## DAVID LINK

## Scrambling T-R-U-T-H Rotating Letters as a Material Form of Thought

"Know that the secrets of God and the objects of His science, the subtle realities and the dense realities, the things of above and the things from below, belong to two categories: there are numbers and there are letters. The secrets of the letters are in the numbers, and the epiphanies of the numbers are in the letters. The numbers are the realities of above, belonging to the spiritual entities. The letters belong to the circle of the material realities and to the becoming." Aḥmad al-B $\bar{n} n \bar{\imath}(\mathrm{~d} .1225)^{1}$

The works of the Majorcan philosopher and missionary Ramon Llull (1232-1316) are commonly and rightly regarded as foundational for the development of Western combinatorics and logic. Circular disks inscribed with the letters from B to K, which can be rotated in relation to each other, play a central role in his


Fig. 1: Alphabetic disk in Ars Generalis Ultima, 1308.

[^0]Ars Inveniendi Veritatem. A working model of the paper machine was included in several of his publications; a thread through the middle held the disks together.
Llull's motives are not easily understood today, but it seems safe to state that in his quest to convert the "infidels" to Christianity, the disk construction served theoretical functions as an encyclopaedia of religious thought, a tool to inspire meditation about its main topics, a way to generate new propositions, or even to abolish language altogether: "The idea is to present in a single book everything that can be thought [...] as well as everything that can be said [...]. From the binary combination of terms in this universal grammar, conceived as general principles, it would be possible to find a solution to any question the human mind can pose. As an art of questioning and getting answers to a variety of matters, it is applicable to all the sciences." ${ }^{2}$
Several authors have pointed out that the model for Llull's disk was a divinatory device called a $z \bar{a} \backslash i r j a b$, which was in use among the Muslims he was seeking to convert. ${ }^{3}$ The artefact offered a most astonishing function: Taking into account the moment in time of the enquiry, it generated a rhymed answer to any question posed. Possibly Llull was impressed by the fact that even kings placed a high trust in the procedure: "Many distinguished people have shown great interest in using [the $z \bar{a} \backslash i r a j a b]$ for supernatural information, with the help of the well-known enigmatic operation that goes with it." ${ }^{4}$
'Abd ar-Rahmān Ibn Khaldūn's Muqaddimab (1377), the introduction to a universal world history in seven volumes, presents a detailed and complete instance

[^1]of the procedure executed on the "Zā ${ }^{\prime}$ irajab of the world" (as one of the translators into English, Franz Rosenthal, spells it). ${ }^{5}$ The device is attributed to the Maghribi Sufi Muḥammad b. Mas-ūd Abū l—cAbbās as-Sabtī, who lived in Marrakech at the end of the twelfth century. ${ }^{6}$ A footnote in the first printed edition of the work in Arabic from Bulaq near Cairo relates that Ibn Khaldūn derived the knowledge about the artefact "from people who work with the $z \bar{a}$ 'irajah and whom [he had] met". ${ }^{7}$ An untitled manuscript by Shaikh Jamāl ad-dīn Abd alMalik b. Abd Allāh al-Marjānı̄ in the library of Rabat, Morocco, establishes it was he who introduced Ibn Khaldūn to the procedure in Biskra, now Algeria, in 1370/1371 (the year 772 A.H.). ${ }^{8} \mathrm{Al}-\mathrm{Marjā} n \overline{1}$ had heard from the $q \bar{a} d \bar{\imath}$ (judge) of Constantine, today also Algeria (who in turn had allegedly been informed by one of the companions of the prophet, Hudhaifa b. Yamān), that the $z \bar{a} ’ i r j a b$ was a traditional and ancient science. ${ }^{9}$ When Ibn Khaldūn questioned the correctness of the proposition, the shaikh suggested simply to ask the device itself:
الزايرجة علم مددث أو قديم
"Az-zā’irja 'ilm mubdath au qadīm" - "[Is the] zā’irja [a] recent or [an] ancient science?" ${ }^{10}$

When the self-referential question was posed, the ascendant (the sign of the zodiac cycle rising at the eastern horizon) stood in the first degree of Sagittarius. Al-Marjānī performed the complicated procedure and explained it to Ibn Khaldūn. It yielded the answer "The Holy Spirit will depart, its secret having been brought forth / To Idrīs, and through it, he ascended the highest summit"

[^2]10 The single letters of the question are: " 1 z ' $\mathrm{yrjt} /{ }^{\prime} 1 \mathrm{~m} / \mathrm{mh} \mathrm{dth} /$ ' $\mathrm{w} / \mathrm{qdym} \mathrm{m}$ ".
("Tarūbanna rūbu l-qudsī ubriza sirrubā / Li-Idrīsa fa-starqā bi-bā murtaqā l-‘ulā") and thus confirmed the information the shaikh had given. The Qur'anic sage Idrīs is commonly identified with the biblical Enoch and fulfils a function similar to the one described by Michael Thompson as "Father of all Tiv": the mythical ancestor at the absolute limit of memory. ${ }^{11}$
The manuscript from Rabat differs slightly regarding the time and reveals an unexpected trait of the learned lawyer (faqī): "Then [after the answer had been obtained], [Ibn Khaldūn] started to dance and to turn on the terrace of his house. The ascendant of the question was the 18th [degree] of Sagittarius." ${ }^{12}$ The difference of 17 degrees equals slightly more than one hour, since the sign at the eastern horizon progresses by one degree every four minutes. If the time indicated refers to the moment when Ibn Khaldūn started to dance, the entire explanation took place in just 68 minutes, a very short time considering the operation's complexity.

## 1. Numeral Systems

The algorithmic artefact of the zā'irja, constructed from paper and driven by a human computor, processed symbols guided by rules. The symbols originated from three different alphabets, sets of signs that seamlessly converted between letters and numerals and belonged to two principally different ways of counting. During the period under consideration, from 1000 to 1400 , two pre-positional and one positional system co-existed and the transition to the latter was about to take place.
The 28 symbols of the Arabic alphabet numerically represent units, tens and hundreds up to thousand, without place value and zero (cf. Figure 2). They were called burūf al-jumal ("letters for summing up, collecting"), or abjad, alluding to the four characters at the beginning. ${ }^{13}$ The latter name derived from the mnemonics employed to memorise the numerical order of the signs, which was different from the alphabetic sequence after the symbols were rearranged by similarity in the seventh or eighth century. "Abjad" is the first of eight such words in Eastern Arabia, the others being "hawazin", "ḥuṭiya", "kalamuna",

11 Muqaddimah, trans. Rosenthal, vol. 3, p. 213, fn. 921 [III, 178]; cf. Michael Thompson, Rubbish Theory. The Creation and Destruction of Value (Oxford, 1979), pp. 65-69.
12 Renaud, Divination, p. 215.
13 Edward W. Lane, An Arabic-English Lexicon [1863-1893] (Beirut, 1968), vol. 2, p. 459 f.
"s'afaṣ", "qurshat", "thakhudh" and "dazaugh", which were later thought to represent archaic kings, demons, or the days of the week. ${ }^{14}$

| 1 | ب | T | د | - | ง | j | $\tau$ | b | ي | $\leq$ | $J$ | - | ن | $ص$ | $\varepsilon$ | ف | ض | ق | , | س | ت | ث | $\dot{\tau}$ | j | b | $\dot{\varepsilon}$ | ش |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| , | b | j | d | h | w | z | h | t | y | k | 1 | m | n | s | ، | f | d | q | r | s | t | th | kh | dh | z | gh | sh |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 |

Fig. 2: Arabic letters, Western numerical interpretation. ${ }^{15}$

The zimām symbols follow the same system, but only designate numerals and do not contain a character for thousand (Figure 3). They translate easily into Arabic letters of the same value. In both sets of signs, composite numbers are formed by joining together the respective symbols for units, tens, hundreds, and so on; in the first decreasing in value from right to left (the "natural" writing direction), and increasing in the second. ${ }^{16}$ Hence 333 would be represented as سلـج in abjad, or $6{ }^{6}$ in zimām. ${ }^{17}$


Fig. 3: Zimām numerals, according to Rosenthal (1958).

14 Georges Ifrah, The Universal History of Numbers. From Prehistory to the Invention of the Computer (London, 1998), p. 243f.
15 The Arabic Orient and Occident match letters and numbers slightly differently; cf. Muqaddimah, trans. Rosenthal, vol. 3, p. 173, fn. 809. Throughout this article, alif will be transliterated as ' and 'ayn as '. 16 Azzedine Lazrek, personal communication, 14 May 2008.
17 The symbols employed in this paper, slightly different from the ones below, which Rosenthal copied from one of the manuscripts of the Muqaddima, were published by Azzedine Lazrek, Cadi Ayyad University, Marrakech, Morocco, as part of a proposal to include them in the Unicode standard; cf. A. Lazrek, Unicode-Proposals symbols for Unicode Consortium, 17 March 2008. http://www.ucam.ac.ma/fssm/rydarab/unicode.htm. The proposal was accepted on 25 April 2008, and the characters will probably be allocated in the range 10E60..10E7E; cf. Unicode Consortium, Proposed New Characters—Pipeline Table, 1 May 2008. http://unicode.org/unicode/alloc/Pipeline.html.

Georges S. Colin has shown that the genesis of the 27 signs can be traced back to one of the counting systems of antiquity. In addition to the 24 letters of their alphabet, the Greeks adopted three obsolete characters from their archaic script, $\delta$ (wau or digamma, $=6$ ), $\mathrm{Q}(\mathrm{koppa},=900)$, and $\lambda(\mathrm{sampi},=90)$, and employed the resulting 27 symbols to designate units, tens, and hundreds. The Copts, Christianised Egyptians within the Byzantine Empire, used them from the third century onwards to calculate the finances of state. When Arabs conquered the country in 640, the Copts remained officials, but since the new governors of Egypt did not want the state finances exclusively in Christian hands, in 706 they obliged them to perform their work in the Arabic language, and put Muslim functionaries at their side. The Copto-Greek numerals that they continued to use probably entered the Islamic world via this route. It was not until the beginning of the nineteenth century that accounting with the esoteric numbers was finally forbidden in Egypt. ${ }^{18}$
In zimām, units, tens, and hundreds were designated by different characters and kept apart as qualitatively dissimilar. The highest value that could be represented by placing horizontal strokes below the signs was $999,999 .{ }^{19}$ An example is appropriate here to illustrate the clumsiness of such systems: the addition of 222 and 444 in Roman numerals, CCXXII + CDXLIV, which is trivial in "Arabic" signs. First, a conventional hindrance, the subtractive notation in the second term, must be removed, which results in CCCCXXXXIIII. Next, the addition can be performed by grouping identical symbols together. The sorting produces CCCCCCXXXXXXIIIIII, which needs to be converted to standard form by reducing the signs to the highest values, DCLXVI, 666. Obviously, more complex arithmetical operations cannot be performed in this way, which is why they were not executed with the numbers themselves, but with a separate artefact, the abacus. As only the results were written down, early Arabic mathematical treatises very rarely contain corrections and no intermediate calculations at all. ${ }^{20}$ The device's speed is due to the mechanical performance of several simple operations. ${ }^{21}$

[^3]One of the main procedures on the reckoning-board is the scaling of values, where one bead is assumed to represent fives, tens, and so on. When a number needs to be carried over to the next counter, a second operation is employed, the calculation of the remainder after division through a certain quantity, 5 in the case of this device. It evolves naturally when a group of units needs to be compressed into a sign with a higher value, and may derive from the cyclical character of time and the practice of clock-building. The modern term is modulo. In


Fig. 4: Ancient Greek abacus found on Salamis, fifth to fourth century B.C. ${ }^{22}$

21 Most basic calculations are executed faster on an abacus than with an electric calculator, as proven by a contest held in Japan shortly after the Second World War, in which the soroban (Japanese abacus) champion won 4 to 1 against his North American counterpart; cf. Ifrah, Universal History, p. 289f.
22 The tablet measures approximately $149 \times 75 \times 4.5 \mathrm{~cm}$. The right side of the device represents the number $9,823,1(\times 5000)+4(\times 1000)+1(\times 500)+3(\times 100)+0(\times 50)+2(\times 10)+0(\times 5)+3(\times 1) ; \mathrm{cf}$. Cajori, Mathematical Notation, p. 22f., and Ifrah, Universal History, pp. 201-203, for a more exact treatment. A similar table is already mentioned in the Athenian Constitution (written between 332 and 322 B.C.): "And when all have voted, the attendants take the vessel that is to count and empty it out on to a reckoning-board ["ábaka"] with as many holes in it as there are pebbles, in order that they may be set out visibly and be easy to count, and that the perforated and the whole ones may be clearly seen by the litigants. And those assigned by lot to count the voting-pebbles count them out on to the reckoningboard ["ábakos"], in two sets, one the whole ones and the other those perforated." Aristotle in 23 Volumes, vol. 23: Athenian Constitution, trans. H. Rackham (Cambridge, MA, 1952), sect. 69. 1.

Figure 5, the addition of $3+3=6$ is performed on the abacus as an example. While the lower beads represent the units, the upper ones possess a value of five, and both are only counted if they touch the centre. If three is to be added to the three entities already there, five needs to be carried over to the upper part, because there are not enough of them in the lower. After the operation, one is left in the units-the remainder when six is divided by five. ${ }^{23}$


In contrast to zima $\bar{a} m$, the ghub $\bar{a} r$ system represents numbers of any magnitude by arranging only ten signs from 0 to 9 (including zero) in a positional manner, which is comparable to the system most probably invented by Indian mathematicians and still employed today. ${ }^{24} 333$ would be expressed as $\{\xi \xi$-economically superior and mathematically momentous-using the same character to indicate units, tens, and hundreds.

The German naturalist and explorer Alexander von Humboldt had already pointed out in the nineteenth century how close the ghubār numerals were to the


Fig. 6: Ghubār numerals, according to Rosenthal. ${ }^{25}$

[^4]modern positional system by quoting his "friend and teacher" Sylvestre de Sacy, who had discovered them in a manuscript in the library of the old Abbey of St. Germain-des-Prés: "The gobar has a strong relation with the Indian number, but it does not possess a zero. ${ }^{" 26}$ Humboldt, however, believed zero was expressed by marks placed above the signs: tens were indicated by one, hundreds by two, and thousands by three dots in superscript, constituting a "hybrid" counting system. ${ }^{27} \mathrm{He}$ speculated that it was these marks that had given the numbers their "strange name gobar or dust writing". ${ }^{28}$

Literally, "ghubār" (غبار) does indeed signify "dust", not in the sense of the miraculously appearing house dirt of our days, but rather the finest possible kind of sand. ${ }^{29}$ This led many scientists in the nineteenth century to believe that the name derived from an ancient means of calculation. Since antiquity, a board of wood covered with dust (called "lawha" in Maghreb) had been used for this purpose, writing numbers with a stick on it and deleting others in a performative kind of arithmetic. ${ }^{30}$ It complemented the wax tablets of Aristotelian fame. This setup allowed relatively flexible processing of data, unlike working with clay or stone, and could be described as volatile "random access memory", since its fluidity permitted the modification of any grain on the surface at any time. ${ }^{31}$
known documents which refer to Ghubar numerals and calculation date back to 874 and 888 CE" and were apparently found in India (p. 536).
26 Translation D.L. De Sacy also translated parts of the Muqaddima; cf. Baron Antoine Isaac Silvestre de Sacy, Extraits de Prolégomènes d'Ebn Khaldoun, in: Relation de l'Égypte, par Abd-Allatif, Médecin Arabe de Bagdad, trans. S. de Sacy (Paris, 1810; reprint Frankfurt am Main, 1992), pp. 509-524 (translation vol. 5, chap. 4 and vol. 4, chaps. 3 and 4), pp. 558-64 (Arabic text), and 'Abd ar-Rahmān alJāmī, Vie des Soufis ou Les Haleines de la Familiarité, trans. S. de Sacy (Paris, 1831; reprint Paris, 1977), pp. 16-20 (Arabic text), pp. 20-27 (translation "Du Sofisme").
27 In this representation, the number is decomposed into units, tens, hundreds, and so on, and then each is written down in two parts in a multiplicative manner. 7,659 would be expressed as $7^{000} 6^{00} 5^{0} 9$, $7 \times 1000+6 \times 100+5 \times 10+9 \times 1$. The abbreviation of such systems led to positional notation very early on; cf. Ifrah, Universal History, pp. 330-340.
28 Alexander von Humboldt, Über die bei den verschiedenen Völkern üblichen Systeme von Zahlzeichen und über den Ursprung des Stellenwerthes in den indischen Zahlen. Crelle's fournal für die reine und angewandte Mathematik 4 [3] (1829): 205-231, esp. 213, 222-224. In the manuscript that Rosenthal took the gbubār numerals from, zero is placed to the right side of the sign; cf. the ten in Figure 6.
29 Lane, Arabic-English Lexicon, vol. 6, p. 2224.
30 Abdeljaouad, Manuscrit mathématique, p. 20; Ifrah, Universal History, pp. 207-209. To learn how computation was performed on it, see pp. 556-560.
31 Real volatile storage only becomes possible when the medium employed is even more malleable, like water or electricity; cf. D. Link, There must be an angel. On the beginnings of the arithmetics of

Conversely, Solomon Gandz showed in the 1930s that burūf al-gbubār did not indicate "letters of dust", but rather "signs employed on the abacus", and that the word consciously reflected the etymology of Greek ' $\dot{\alpha} \beta \alpha \xi$ derived from Hebrew אבק (abaq - "dust"). ${ }^{32}$ The first implementation of the technical form allegedly consisted in lines in the sand on a tablet and pebbles moved around on them, hence the name.
The Muqaddima of 1377 is the earliest document that mentions the utilisation of ghubār in North Africa. Apart from proving that the symbols were known by Muslim fortune-tellers, Ibn Khaldūn states they were "used for numerals by government officials and accountants in contemporary Maghrib", probably already for some time. ${ }^{33}$ Only in the sixteenth century did the system become more common in Morocco, probably via Spain. Colin argues that even if the ghubār numerals are found in the Muslim Occident from the middle of the tenth century, their use was limited to mathematical calculations of Indian-type arithmetic. In everyday situations, the non-positional symbols were retained, and the two systems coexisted for several hundred years. ${ }^{34}$ At the end of the seventeenth century, the ghubār began to replace the Greek numbers, and seventy years ago, they were only still employed in one city of Morocco, Fes, among notaries. Although the distribution of inheritances was calculated in a positional system, it was fixed in zimām, in the hope that the "quasi-cryptographic" symbols would prevent the manipulation of the result by laymen. ${ }^{35}$
From the moment a positional system that includes zero is employed, the abacus is annulled, conserved, and elevated ("aufgehoben" in German) in the numbers. The technical form is transcribed into a more abstract and flexible one, which unlike previous schemata enabled the explosive development of arithmetic.

[^5]
## 2. Material Forms of Thought

The operational basis of the $z \bar{a}{ }^{\prime} i r j a$ consists in three parts: two tables and a poem. The first and main element, the "front", shows a zodiac circle divided into eight concentric subsections with twelve chords (awt $\bar{a} r$ ) extending from the centre outwards, which carry signs from the different alphabets (see Figure 7 and for a transliteration, Figure 8).


Fig. 7: Zā'irja front from a Turkish manuscript [~1415] of the Muqaddima. ${ }^{36}$

[^6]A rectangular table of 55 vertical squares times approximately 131 horizontal squares, most of them also marked with symbols from the three alphabets, forms the second part of the system, the "back" (see Figure 10). ${ }^{37}$


Fig. 8: Transliteration by Rosenthal. ${ }^{38}$

37 The number specified in the text, 131, differs from the actual layout of the artefact. Rosenthal counted the quantity of horizontal squares in the Turkish manuscripts as 128 (Muqaddimah, trans. Rosenthal, vol. 1, p. 240, fn. 370), whereas the one he presented as a facsimile and also the one in the Cairo edition of 1957-1962 contains 129 columns (vol. 4, p. 1168).
38 Muqaddimah, trans. Rosenthal, vol. 3, end pocket.


Fig. 9: Zā'irja front from another [~1394] Turkish manuscript of the Muqaddima (MS Damad Ibrabim 863, Süleymaniye library, Istanbul, p. 352).


Fig. 10: Reverse of the zā’irja in the Turkish manuscript. ${ }^{39}$

39 MS Ahmet III, 3042, vol. 1, fol. 237. Numbering of rows and columns is mine, D.L.


The third artefact needed to perform the procedure is an immaterial form of thought. The "key poem", ascribed to the Sevillian scholar Mālik b. Wuhayb, is entirely composed in the signs of one alphabet, the Arabic letters. ${ }^{40}$
سؤال عظيم الخلق حزت فصن اذن غرائب شك ضبطه الجد مثلا



Fig. 11: Reverse of the zā’irja in MS Damad Ibrahim 863 (Sïleymaniye library, Istanbul, p. 353f.).

Rosenthal translated the signs "s w'l/'zy m/'lkhlq/h zt/fṣn/'dhn// ghr'yb/shk/ḍth /'ljd/m thl'" - "su'al azym alkhalq hazat fasun odhun/ gharāyb shak dabtuhu eljid mathalan" as "A weighty question you have got. Keep then, to yourself / Remarkable doubts which have been raised and which can be straightened out with diligence". ${ }^{41}$ His successor Abdesselam Cheddadi rendered the poem as follows: "You possess the question of the grand natural form. Thus conserve the strange doubts that have been raised and which the diligence can dissipate. ${ }^{"+2}$ The verse seems especially difficult to translate because it only represents a mnemonic aid, like the abjad words above, to remember otherwise meaningless sequences of characters. In Arabic chronograms, poems whose letter values reveal the date of an important event, a similar technique has reached the realms of literary art. ${ }^{43}$

## 3. The Archaeology of Algorithms

At the beginning of the detailed description of the procedure on the $z \vec{a} \vec{a}^{\prime} i r j a$ in the third volume, ${ }^{44}$ Rosenthal wrote in a footnote:
"The letters evolved in the procedure described by Ibn Khaldūn are marked in this translation by boldface type. However, the rationale of their determination and the relationship of the description to the table are by no means clear to me. As in the case of the $z \bar{a}$ 'irja poem, a translation-one might rather call it a transposition of Arabic into English words-is offered here in the hope that it may serve as a basis, however shaky, for future improvement." ${ }^{45}$

[^7]Rosenthal seems to have devoted considerable time and effort to understand the operations, as advocated by various footnotes tentatively trying to establish more consistency. Unfortunately, his papers have not yet been made accessible to the public. ${ }^{46}$ Vincent Monteil, the translator of the French edition, commented:
"The long 'explication' given by Ibn Khaldūn is-at least for us, humans of the twentieth century-remarkably confused. I have engaged myself in the succession of Rosenthal, profiting from his commendable efforts and the recourse to the tables that Slane could not have consulted. But the difficulties of translation are at times insurmountable, at least with the current state of our knowledge." ${ }^{47}$

Rosenthal employed the classical tools of philology to reconstruct the exact wording of the passage in question and the letters on the tables. Due to a less cautious attitude, Monteil again slightly corrupted the text in his translation. ${ }^{48}$ Both failed to establish the concrete procedure executed on the device. The "Grand-Druide" Gwenc'hlan Le Scouëzec, a collector of practical knowledge of all kinds of esoteric systems, wrote in the "Dictionnaire des Arts Divinatoires": "The usage of $Z \bar{a}$ 'irja consultation seems to have vanished, or, at least, to have been completely forgotten in the Maghreb. [...] Thus, it seems that the $Z \bar{a}{ }^{\prime} i r j a$ has fallen into more or less complete desuetude." ${ }^{49}$

Yet this pessimistic statement does not seem to be entirely justified. An Internet search of the letter sequence زايرجة (zä’irja) produced 804 results, among which, apart from Arabic editions of the Muqaddima, were several postings in newsgroups. These results show that at least some variant of the procedure is

[^8]still used, for example, to calculate the answer to the question "Will America strike Iran this year?", which was computed in 2007. The answer was: "Yes, they will strike, but America will thereby be destroyed". The second enquiry, "When will America strike and who will win the war?" yielded "The war nears, the winner will be the Persians" (Iran today). ${ }^{50}$

The present chapter will attempt to rely on the "shaky ground" that Rosenthal has the merit to have established, and to reconstruct the routine employed to generate the answer. The difficulty of the classical disciplines of historiography and philology to deal with the complexity of algorithms and the devices they are embodied in demonstrates that their archaeology necessitates a different approach. The methods of cryptology offer an important aid in the reconstruction of regularity in mutilated or scrambled symbol systems. Their applicability to a wider range of questions can be seen from the fact that they are increasingly employed in genomics. ${ }^{51}$ Through an analytical effort, the archaeology of algorithms and their artefacts aims at winning back as much ordered structure as possible from entropy, like the general discipline it belongs to.

## 4. The Signs of the Moment

In the first step of the procedure, the letters are taken off three of the chords on the $z \bar{a} ’ i r j a$ 's front, depending on the ruler and degree of the ascendant in the moment of the question. The sign rising at the eastern horizon wanders through the 360 degrees of the zodiac circle within a day. At the time, the data might have been obtained by consulting astronomical tables or by employing an astrolabe, an instrument closely resembling the one discussed here. ${ }^{52}$ The

[^9]description in the Muqaddima reads: "The ascendant is in the first degree of Sagittarius. Thus, we place the letters of the chord of the beginning of Sagittarius and the corresponding chord of the beginning of Gemini and, in the third place, the chord of the beginning of Aquarius up to the limit of the centre."53 The degree and, consequently, the letters selected change every four minutes. Because the ascendant in the moment of the operation was Sagittarius, the characters are taken off the respective chord, following it over the centre to the opposite, Gemini. Then, the symbols bordering the third sign from the ruler are noted. ${ }^{54}$ Unfortunately, Ibn Khaldūn does not mention if and how this relates to the current degree.
Even though the procedure at first sounds clear and simple, one encounters considerable difficulties to match the 73 letters listed in the text as " $\mathrm{s}, \mathrm{t}, \mathrm{d}, \mathrm{t}, \mathrm{h}, \mathrm{n}$, th, k, h, m, ḍ, s, w, n, th, h, s, alif, b, l, m, n, s, ‘ayn, f, d, q, r, s, y, k, l, m, n, s, ‘ayn, $\mathrm{f}, \mathrm{q}, \mathrm{r}, \mathrm{s}, \mathrm{t}, \mathrm{th}, \mathrm{kh}, \mathrm{dh}, \mathrm{z}, \mathrm{gh}, \mathrm{sh}, \mathrm{t}, \mathrm{k}, \mathrm{n}$, 'ayn, h, s, z, w, h, l, s, k, l, m, n, s, alif, b, j, $\mathrm{d}, \mathrm{h}, \mathrm{w}, \mathrm{z}, \mathrm{h}, \mathrm{t}, \mathrm{y}$ " with the chords they originate from. ${ }^{55}$ This is due to several ambiguities and complications, which apparently none of the translators were able to overcome. First, it is unclear if the "beginning" of a sign is the chord on the left or the right side. Second, the text seems to imply that the letters are read following Sagittarius inwards over the centre, then along Gemini outwards, and finally Aquarius inwards again. Third, it is not easy to decide which of the numbers and letters on the table are to be considered, some being located close to the chords, but not on them. Additionally, the mixing of the different sign systems adds to the uncertainty and creates an extra layer of obscurity. Several characters can be regarded as Arabic letters or as numerals of one of the alternative conventions, as their forms are sometimes very similar.
The difficulties do not primarily stem from mutilation in the course of historical transmission, which relied for a long time on handwriting. Rosenthal, who consulted most of the Turkish copies of the work, commented:
"All these manuscripts have the same textual value that, in the period after the invention of printing, would be ascribed to a book printed under its author's

52 Cf. Arianna Borrelli, Aspects of the Astrolabe. "Architectonica Ratio" in Tenth- and Eleventh-century Europe (Stuttgart, 2008), p. 65: "At the same time, though, the astrolabe was also a tool for astrologers, and astrology played an important role in tenth-, eleventh- and twelfth-century Arabic-Islamic court life."

53 Muqaddimah, trans. Rosenthal, vol. 3, p. 199 [III, 163].
54 It is actually the second, because the starting field is counted as 1 .
55 Muqaddimah, trans. Rosenthal, vol. 3, p. 211 [III, 176].
supervision. There may be occasional mistakes, but a carefully written manuscript usually compares favourably with a printed text. Most manuscripts of this type may be confidently regarded as authentic copies of the text, and any factual mistakes or miswriting they contain may be considered the author's own. Under these circumstances, we should expect the variant readings to be comparatively few and insignificant. Collation shows this to be, indeed, the case." ${ }^{56}$
It will be seen below that the section on the $z \bar{a} \bar{a}^{\prime} i j a$ does actually contain errors of transcription, but also of interpretation. This is due to the very special character of the chapter in question, and the almost complete arbitrariness-the absence of discernable rules the signs would be regulated by-of its basis, the two tables. The words describing the tables are determined by a dictionary, which only assigns meaning to some of them and, consequently, allows reconstruction in case of necessity. If no such order is employed, the only remaining system of rules is the alphabet, which correlates single letters and numerals in a certain sequence. The situation is further aggravated by the astonishingly high complexity of the operation, given the period in which it was invented. Lewis Carroll once illustrated its effect in the following way: "Can you do addition?" the White Queen asked. 'What's one and one and one and one and one and one and one and one and one and one?' 'I don't know' said Alice, 'I lost count.'" 57 Obviously the rather sceptical Ibn Khaldūn, who was courageous enough to judge the language of Qur'ân as defective, ${ }^{58}$ was impressed by the procedure as a serious method of enquiry guided by strict rules:
"Many people lack the understanding necessary for belief in the genuineness of the operation and its effectiveness in discovering the object of enquiry. They deny its soundness and believe that it is hocus-pocus. The practitioner, they believe, inserts the letters of a verse he (himself) composes as he wishes, from the

[^10]letters of question and chord. He follows the described technique, which has no system or norm, and then he produces his verse, pretending that it was the result of an operation that followed an established procedure. This reasoning is baseless and wrong.[...] In order to refute this [...], it is sufficient for us (to refer to the fact) that the technique has been observed in operation and that it has been definitely and intelligently established that the operation follows a coherent procedure and sound norms." ${ }^{59}$

## 5. Fixing the Chords

The number of the signs from the chords mentioned in the text is 73 , which can be reached in only two ways, assuming the information about the zodiac they originate from is not completely wrong: Either its beginning is located on the right, and all symbols are taken into account, which results in 73 characters. Or the left is looked up, then numbers have to be disregarded, because otherwise there are too many signs-93. If only letters are counted the result is 72. To find out which of the alternatives is the correct one, and, consequently, which of the chords the characters originate from, a classical tool of cryptanalysis has been employed. A frequency count of the letters given in the text ("T") is compared to that of the two possible sets of symbols on the $z \vec{a} ’ i r j a$, the left-hand ("L") and the right-hand ("R") alternative. ${ }^{60}$ Figure 12 presents the results.

|  | , | b | j | d | h | w | z | h | t | y | k | l | m | n | s | ‘ | f | d | q | r | s | t | th | kh | dh | z | gh | sh |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T$ | 2 | 2 | 1 | 2 | 4 | 3 | 2 | 3 | 4 | 2 | 4 | 4 | 4 | 6 | 7 | 3 | 2 | 2 | 2 | 2 | 3 | 1 | 3 | 1 | 1 | 1 | 1 | 1 |
| $L$ | 4 | 3 | 3 | 2 | 4 | 3 | 1 | 2 | 2 | 2 | 4 | 4 | 2 | 2 | 3 | 3 | 3 | 1 | 5 | 3 | 2 | 1 | 2 | 1 | 2 | 1 | 3 | 4 |
| $R$ | 2 | 3 | 1 | 2 | 3 | 2 | 1 | 4 | 3 | 2 | 5 | 4 | 2 | 7 | 7 | 1 | 1 | 3 | 5 | 2 | 1 | 2 | 3 | 1 | 1 | 2 | 1 | 2 |

Fig. 12: Calculation of deviation between chord letters in text $(T)$ and on front table, left-hand (L) and right-band $(R)$ alternative.

[^11]Apart from the matching extremes for " n " and " s " (marked in grey), the sum of the squares of deviation equals 79 for the first case, for the second only 34 , and the variance per letter is 2.82 versus 1.21 . This clearly indicates that the "beginning" of a sign is located on the right, following the direction of Arabic script and the orientation of the zodiac circle, which starts with Aries and moves counter-clockwise until the last one, Pisces, is reached. Without frequency analysis, the similarity can hardly be spotted.
The letters are taken off the chords and compared to the sequence found in the text. Apart from a region in the middle, which corresponds to Gemini read outwards, not many characters match, only about $26 \%$ (marked in grey).

| $T$ | s | t | d | t | h | n | th | k | h | m | d | s | w | n | th | h | s | s | b | l | m | n | s |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $C$ | $\mathrm{t}^{400}$ | k | r | q | $\mathrm{d}^{90}$ | b | $\mathrm{~d}^{+}$ | $\mathrm{s}^{60}$ | n | $\mathrm{q}^{100}$ | $\mathrm{l}^{30}$ | b | , | sh | $\mathrm{dt}^{500}$ | $\mathrm{n}^{50}$ | $\mathrm{w}^{6}$ | $\mathrm{~s}^{60}$ | $\mathrm{~d}^{90}$ | $\mathrm{t}^{\mathrm{g}}$ | $\mathrm{h}^{5}$ | $\mathrm{~b}^{2}$ | $\mathrm{th}^{500}$ |


| ، | f | d | q | r | s | y | k | 1 | m | n | s | - | f | q | r | s | t | th | kh | dh | z | gh | sh | t | k | n |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{n}^{50}$ | $\mathrm{h}^{5}$ | z | k | t | s | y | k | 1 | m | n | s | ' |  | q | r | s | t | th | kh | dh | z | gh | sh | y | t | h | z |


| h | s | z | w | h | 1 | s | k | 1 | m | n | s |  | b | j | d | h | w | z | h | t | y |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| w | h | d | j | n |  | s | n | m | 1 |  | $\mathrm{d}^{90}$ | 1 | k | q | ${ }^{60}$ | $h^{8}$ | q | $h^{8}$ | $\mathrm{f}^{\text {so }}$ | $\mathrm{n}^{50}$ | s | ḥ |

Fig. 13: Comparison of symbols in text $(T)$ and from chords (C), read inwards, outwards, inwards. ${ }^{61}$

However, some sequences from above repeat below in the opposite direction, like "k, l, m, n, s, alif" towards the end (framed with thicker lines). If all the chords are read from the centre outwards, the resemblance of the letter groups becomes much closer, $75.3 \% .{ }^{62}$

| $T$ | s | t | d | t | h | n | th | k | h | m | d | s | w | n | th | h | s | , | b | 1 | m | n | s |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | s | t | k | z | $\mathrm{h}^{5}$ | $\mathrm{n}^{50}$ | $\mathrm{t}^{500}$ | $\mathrm{b}^{2}$ | $\mathrm{h}^{5}$ | $t^{\text {t }}$ | $\mathrm{d}^{90}$ | $\mathrm{s}^{60}$ | $w^{6}$ | $\mathrm{n}^{50}$ | $\mathrm{dh}^{500}$ | sh |  |  | b | $1^{30}$ | $\mathrm{q}^{100}$ | n | $\mathrm{s}^{66}$ | ${ }^{60}$ |


|  | f | ¢ |  | q | r | s |  | y |  |  |  | m | n |  |  | f | q | r | s | t | h | h | dh | \% | g | gh | sh | t | k | k |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}^{+}$ | b | d |  | q | r | k | $t^{4}$ | y |  |  |  | m | n |  |  |  | q | r | s | t | h | h | dh | \% |  | gh | sh | $h^{8}$ | $\stackrel{ }{ }$ | S |


| n | ' | h | s | z | w | h |  | 1 | s | k | 1 | m | n | s |  | b | j | d | h | w | z | h | t | y | y |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{n}^{50}$ | ${ }^{\text {so }}$ | $h^{8}$ | $\mathrm{s}^{\text {sol }}$ | q | q | $h^{8}$ | k | 1 | $\mathrm{d}^{\text {90 }}$ | k | 1 | m | n | $\stackrel{ }{\text { s }}$ |  | n | j | d | h | w | z | h | t | y |  |

Fig. 14: Comparison of letters in text and from chords, all read outwards.

As can be seen from the question mark that Rosenthal put at the " k " on the outer end of Sagittarius (see Figure 8), in places he could not decipher the signs unambiguously. Since he did not understand the procedure, he was unable to employ its regularity to correct the mutilation of single letters and had no means at his disposal to verify his transliteration.
The Arabic alphabet consists of approximately only 14 different basic forms, which are further differentiated into the 28 letters by diacritics. Historically, the marks were put above the signs when it had become impossible to tell them apart:
"Certain groups of letter-shapes in the original Semitic alphabet were so simplified in the development of Arabic script that their forms became wholly identical [...]. The degree of confusion created was such that a system of dots had to be introduced in writing Arabic to differentiate letter-shapes that had merged", around the eighth century. ${ }^{63}$
In cursive Arabic script, most of what Western authors would consider the symbol's body is disregarded and the smallest conceivable difference, single dots,
 and " q " (ق) apart. Even if their sound relates the letters, as in the case of "d" (د) and "dh" $(\mathrm{j})$, their numerical value, which matters equally as much in the $z \vec{a}{ }^{\prime} i r j a$ procedure, disagrees, with $\lrcorner$ representing 4 and $j, 700$. An irregularity in the paper or the sprinkling of ink may easily cause a dot to appear, just as bleaching of the material might make one vanish. Such mutilations can be spotted and corrected in case of meaningful text, because the miswritten word does not exist. Knowledge of the language then allows one to suggest a close alternative from the limited set of possible well-formed sequences. Semantically meaningless expressions can only be protected against distortion by repetition or, in cases where they were generated programmatically, by reconstructing the rules guiding them. In this regard the doubling of character sequences can be interpreted as a "null"-algorithm. ${ }^{64}$
Another factor that contributes to the unintended transformation of signs in Arabic originates from the way in which the scribe wrote them on the scroll.

[^12]Because of its fragility, and probably also to avoid smearing the fresh ink when he moved his hand further to the left, the scribe turned the papyrus 90 degrees counter-clockwise and proceeded from top to bottom. Especially with uncommon characters it could happen that he performed the required mental rotation incorrectly. This explanation has been proposed to account for the change of form of the Arabic numerals 2, 3, and 7 around the year 1000. ${ }^{65}$

The reconstruction of the chords the letters were taken from provides cryptanalytically, as it were, a "depth of two", ${ }^{66}$ and allows a correction of Rosenthal's transcription.

## 6. The Transliteration Re-read

In the following three Figures (Figs. 15-17), the chords have been copied from the $z \bar{a}, i r j a$ front table and all places are marked where they disagree with the sequence specified in the text.


Fig. 15: Characters on Sagittarius.

The first differing letter, "k", $s$ (cf. Figure 15 , read from right to left), appears as " d " in the text. Two of the manuscripts consulted by Rosenthal also read " $k$ " at this place of the descriptive sequence. Two similar characters with the right value might have been miswritten, the Arabic letter "d", د, and the ghubār numeral 4, $\boldsymbol{f}$. In general, signs in the text possess a higher weight of evidence because there are more copies of the manuscript than of the $z \bar{a}$ 'irjab table and because some of these symbols constitute the answer, a semantic, and thus regular sequence. In this particular case, however, the letter does not appear in the output, and therefore, the most probable one could only be determined by a collation of all versions of the passage. The second sign marked, " z " (ظ), occurs instead of " $t$ " $(b)$.

[^13]The difference between the two consists in a single dot that might have been added accidentally in the transmission process. Because of their context, Rosenthal interpreted the next two letters, $\leqslant$ and $\boldsymbol{\rho}$, as numerals. In his transliteration, all the signs of the sequence underlined (symbols 5 to 15) were read as quantities: " $5,50,500,2,5,9,90,60,6,50,500$ ". Since he could not match the chords with the text, the only assumption left to him was that symbols from the different alphabets appeared in groups. The comparison of both sequences establishes that the signs in question should be read as letters, "k" and " $m$ ". He omitted symbol 5 altogether, a stroke half the size of the "alif" two positions later. The text suggests that it should be interpreted as a mutilated $\Delta$, "h". At the next letter, again three dots above may have been added accidentally. Removing them results in "s" (س), the sign in the description. The seventh character was interpreted as zimām 100 , $\boldsymbol{e}$, while the explanatory passage shows the similar "m" ( $\boldsymbol{\rho}$ ) at this place, which might have been miswritten. ${ }^{67}$ It is part of the sequence " $1, \mathrm{~m}$, $\mathrm{n}, \mathrm{s}$, 'ayn, $\mathrm{f}, \mathrm{d}, \mathrm{q}, \mathrm{r}, \mathrm{s}$ " in the text, an excerpt of the Arabic alphabet in numerical order, from 30 to 300 . In the transliteration of the $z \bar{a} \backslash i r j a b$, it runs quite differently, "li3 $, \mathrm{q}^{100}, \mathrm{n}, \mathrm{s}^{60}, \mathrm{~d}^{4}, \mathrm{~b}, \mathrm{~d}^{90}, \mathrm{q}, \mathrm{r}, \mathrm{k}, \mathrm{t}^{400 \text { ". While in places an underlying logic }}$ allows the correction of transmission errors, on the other hand this may lead to the erroneous "reparation" of passages without order, and thus to mutilation of the document. It is impossible to say how the rather huge difference at this location came about. Neither can an explanation be offered for the next character (8), which appears as zimām 4 in the transliteration of the chord and "ayn" in the description. The next symbol could again have been transformed by a misplaced dot, and should be read as $(\mathrm{f})$,as advocated by the sequence in the text. The last differing sign (10) is hardly recognisable and Rosenthal even rendered it as two separate letters.
Since some of the characters from the chords are looked up in the course of the $z \bar{a} \backslash i r j a b$ procedure, they are repeated for a third time, which allows further re-establishment of their original sequence. In particular, the last three letters of Sagittarius are chosen. It follows that there need to be exactly three signs at this location and that they must be " q ", " r " and " s ", otherwise the resulting answer would be different (for details of the lookup procedure see Section 8 and Figure 20, below). Probably, the mysterious last symbol was crossed out and corrected, since a "shin" (ش) was placed upside down at the location, a character similar to the required "s" (س, cf. Figure 15). In Arabic mathematics of the period, the

67 The shape of "mīm" in Maghribi Arabic script published by Ifrah looks very similar to the sign encountered on the chord; cf. Ifrah, Universal History, p. 540.
letter, being an abbreviation of "shay" ("unknown"), usually indicated a free variable, much in the way " $x$ " is employed today. ${ }^{68}$


Fig. 16: Characters on Gemini.

Compared to Sagittarius, the symbols on the second chord are astonishingly well-preserved. This can be explained by the fact that they constitute an almost complete ordered sequence, the letters with the values from 10 to 1000 in the jumal system, excluding 80 and 90 . Zimām or ghubār numbers that would need to be converted are absent. The only difference between the signs on the $z \bar{a} ’ i r j a b$ and in the text is the appearance of an " f " at the place indicated with a question mark in Figure 16. This character is mandatory, as it is looked up during the procedure (see Figure 20, below). The scribe might have mistakenly put down the two similar letters " f " (ف) and "q" (ق) by writing only the second on the chords and omitting the first in the erroneous belief that he had already copied it.


Fig. 17: Characters on Aquarius.

Turning now to the differences on the third chord, the first three cannot be accounted for. The next, " $q$ " (4), might have resulted from an added second dot on the " $z$ " ( $j$ ) found in the description, in the same way that the following one, (5), might have been transformed by the accidental appearance of two dots on the " $w$ " (g) that is required. The " $k$ " $(6, \leqslant)$ is missing from the sequence in the text and needs to be removed for the procedure to work. A dot is found on its left, but remains meaningless because the letter form is never modified by diacritics in this position. The confusion of the 60 in the description and the 90 on the chords (7) may be due to the similarity of the uncommon zimām numerals, $\boldsymbol{\varepsilon}$ and $\frac{1}{\varepsilon}$. The eighth mutilation shows a letter in the process of transformation. Rosenthal transcribed it as " $n$ " (ن) because of the dot above it, but two are also located below. The comparison suggests that the only significant diacritic is the

[^14]upper one of these, which results in "b" (ب). The $z \bar{a} ’ i r j a b$ included in the Cairo edition of 1957-1962 supports the newly established characters at several places by providing the same ones, namely for Sagittarius 4, 7, and 9, and Aquarius 8. As can be seen in Figure 18, the above corrections raise the correspondence between the letters on the chords and those in the text to $93.2 \%$.

| $T$ | s | t | d | t | h | n | th | k | h | m | d | s | w | n | th | h | s |  | b | 1 | m | n | s |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | s | t | k | t | $h^{5}$ | $\mathrm{n}^{50}$ | $\mathrm{th}^{500}$ | k | $\mathrm{h}^{5}$ | m | $\mathrm{d}^{00}$ | $s^{60}$ | $w^{6}$ | $\mathrm{n}^{50}$ | $\mathrm{th}^{500}$ | h | s |  | b | $1^{30}$ | m | n | $\stackrel{5}{5}$ |


| f | f | d | q | r | s | y | k | l | m | n | s | ، | f | q | r | s | t | th | kh | dh | z | gh | sh | t | k |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}^{+}$ | f | $\mathrm{d}^{\text {go }}$ | q | r | s | y | k | l | m | n | s | ، | f | q | r | s | t | th | kh | dh | z | gh | sh | $h^{8}$ | s |


| n | ' | h |  | s | z | w | h |  |  | s | k | 1 | m | n | s |  | b | j | d | h | w | z | h | t | y | y |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{n}^{50}$ | $6^{80}$ | h |  | $5^{60}$ | z | w | h | ${ }^{8}$ | 1 | $\stackrel{5}{60}^{60}$ | k | 1 | m | n | s | , | b | j | d | h | w | z | h | t | y | y |

Fig. 18: Comparison of letters in text and from chords after correction.

Except for the problems of interpretation mentioned above, the inconsistencies already exist in the Turkish manuscripts that Rosenthal consulted, one of which was signed by the author himself and evidently written under his supervision. We do not know how far the historian and learned lawyer Ibn Khaldūn was able to follow the intricacies of the procedure. It is said that letter magic takes a long time to learn and the Kabbalistic tradition goes so far to demand that one should have reached the age of 40 , or even 50 , before studying it. ${ }^{69}$

## 7. The Numbers of the Cycles

After the characters have been taken off the chords, two numbers are calculated from the moment of the question, the first degree of Sagittarius, which in Arabic astrology is regarded as the fourth sign, counting from the end of the zodiac and not from its beginning. ${ }^{70}$ The "principal cycle" (also called the "greatest cycle"

[^15]or "greatest base"), which always equals 1 , is added to the degree, 1 , and the result multiplied by the "ruler": $(1+1) \times 4=8 .^{71}$ If the number obtained is greater than 12 , its modulo is taken. Since the degree ranges from 1 to 30 and the sign from 1 to 12 , this first value lies between $(1+1) \times 1=2$ and $(30+1) \times 12$ $=372$, but will always be scaled to fall within the interval of 1 to 12 . The second number of the moment is calculated by adding the two astrological components, $1+4=5$, and will generally be located between 2 and 42 . Now the reverse of the $z \bar{a}{ }^{\prime} i r j a h$, the rectangular table, comes into play:
"The result of multiplying the ascendant and the greatest cycle by the ruler of Sagittarius [...] is entered at the side of eight. A mark is put upon the end of the number. The five that is the result of the addition of the ruler and ascendant is what is entered on the side of the uppermost large surface of the table. One counts, consecutively, groups of five cycles and keeps them until the number stops opposite the fields of the table that are filled" with "one of four letters, namely, alif, $b, j$, or $z " .{ }^{72}$
The rightmost column of the table only contains eights (see Figure 10), and this is why the text refers to it as the "side of eight". The first value of the moment, 8 , is entered here, moving from the bottom upwards. Then one counts in cycles of 5 , the second number, until I,$ب$, $\boldsymbol{e}$ or is found, most probably from right to left. In the example, three of them pass before $I$ is hit. It is regarded as the result, and squared, 9 , which constitutes the number of the first cycle.
The horizontal counting is very probably performed in the row reached by the vertical movement, and then, the top is looked up as "opposite", as the characters I, 1 and make up most of it. While $ب$, is written in normal orientation here, $\underset{\sim}{\text { a }}$ and $j$ are presented upside down as * $\boldsymbol{*}$ and $\boldsymbol{\varrho}$ respectively, which further adds to the obscurity. Using the table from the Turkish manuscript, the procedure cannot be consistently performed. However, the location of the sign can be deduced, because shortly afterwards 9 is entered in the same line, starting from the previous position:
"The number in the first cycle, which is nine, must be entered in the front [...] of the table adjacent to the field in which the two [the 'side of eight' and the 'uppermost large surface'] are brought together, going towards the left, which is (the field of) eight. It thus falls upon the letter lām-alif, but no composite letter ever comes out of it. It thus is just the letter $\mathbf{t}$ - four hundred in zima $\bar{m}$ letters." ${ }^{73}$

[^16]Rosenthal remarked in a footnote that the numeral for $400, \boldsymbol{y}$, looks similar to the ligature lām-alif, 8 . In the eighth line, this sign is only found on positions 25 , 26 and 27. Counting back 9 from this place, columns 16 to 18 are hit. The ghubār numeral 8 is located in the seventeenth, with an "alif" at the top, as required by the procedure. The section just quoted seems inconsistent, given it is the rightmost column of the table that is called the "side of eight". What is probably meant is that the counting starts next to the field that contains 8 , going towards the left. How this location is reached "leaving three cycles behind" remains a mystery, especially because "alif" is already found five signs before, in column 12.
The complete operation takes place in twelve principal parts-as many as there are signs of the zodiac on the front of the $z \bar{a} i$ irjab. Their numbers are calculated by adding the first value of the moment, 8 , to the quantity obtained in the previous section of the procedure and taking its modulo 12 if it is greater than 12. In the example, this results in $(9+8=17 ; 17 \bmod 12=5$ for the second, $(5+8=13 ; 13 \bmod 12=\mathbf{1}$ for the third, $(1+8=) \mathbf{9}$ for the fourth cycle, and then the sequence repeats. The periodicity depends on the two values of the stellar constellation. A start with 11 and an increment of 7 , for example, leads to the unique series $11,6,1,8,3,10,5,12,7,2,9,4 .^{74}$
Within each of the twelve cycles, between two and six letters are calculated. Each half brings forth 20 symbols, the first triple of each 9 and the second, 11, which exhibits a certain regularity. They are followed by three "results" that generate a further seven. Accordingly, 47 letters are calculated in total (see Figure 19).

| I(9) | II(5) | III(1) | IV(9) | V(5) | VI(1) | VII(9) | VIII(5) | IX(1) | X(9) | XI(5) | XII(1) | R1(9) | R2(5) | R3(1) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 3 | 4 | 6 | 3 | 2 | 2 | 4 | 3 | 3 | 3 | 5 | 3 | 3 | 1 |
| 9 |  |  | 11 |  |  | 9 |  |  | 11 |  |  | 7 |  |  |
| 15 |  |  |  | 11 |  |  |  | 14 |  |  |  | 7 |  |  |
| 20 |  |  |  |  |  | 20 |  |  |  |  |  | 7 |  |  |
| 47 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Fig. 19: Number of letters generated in each cycle, triple, quadruple, and first and second balf.

[^17]The following operations are performed in the course of the procedure:

- Lookup of letters in the signs of the three chords from the front of the $z \vec{a} \overrightarrow{i r j a}$;
- Selection and conversion of symbols in the key poem;
- Generation of characters using the table on the reverse side;
- Scaling of values and miscellaneous arithmetic operations.

These will be analysed in detail in the following sections.

## 8. Chord Letters

In the selection of letters from the chords, the counting always starts from the beginning. The symbol hit is crossed out or otherwise marked and left out of subsequent calculations, which stochastically compares to an urn into which lots selected are not put back. ${ }^{75}$ However, the procedure only works out as described if a certain number of letters is removed in every cycle. Since Rosenthal could not carry out the routine, he translated the deletion of signs in a way that is difficult to understand; only at the beginning are letters from the chords "dropped" (سقط - saqaṭ), which somehow indicates their removal, at all later places they are only "picked" (ضرب - ḍarab). ${ }^{76}$ The characters are crossed out starting from the front, neglecting the ones that have already been looked up.


Fig. 20: Employment of chord letters.

[^18]Figure 20 depicts the lookup process in this part of the system for the complete procedure. Letters and strokes are numbered in the sequence of operations, and a second index shows the value leading to the sign. 13 is first converted with the help of the chords. The letters are counted from the beginning, and " w " in the second row is hit. In Figure 20 it was marked with 1, because this is the first step of the procedure, and with 13 , the input value. A short time later, 17 is entered. Since "w" is now omitted, the "alif" in the second line is reached. It was annotated with 2 (second operation) and 17 (its position). Next, "[o]f the letters of the chords, one drops three". ${ }^{77}$ The first three characters, s, t and k , are crossed out in the third step. Consequently, the 13 that is looked up next hits a different sign, "s" in the second line (marked with 4 and 13). The procedure in the chords can be comprehended by following the numbered order in Figure 20. If the letter sequence recovered in Section 6 is used, the routine works completely as detailed in the text. The only inconsistency occurs at the fourth character in the seventh line, "alif". It is apparently looked up twice, which disagrees with the way the procedure is otherwise executed, omitting letters that were hit before. Seventeen symbols are generated from the chords, and 26 dropped. Figures 21 and 22 show their distribution in the different cycles, with no discernable regularity.

| I | II | III | IV | V | VI | VII | VIII | IX | X | XI | XII | R1 | R2 | R3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| w | , | s, b | r, s | t, q | r | , |  | 1 |  | h | f, s | ', m |  | 1 |
| 4 |  |  | 5 |  |  | 2 |  |  | 3 |  |  | 3 |  |  |
| 6 |  |  |  | 4 |  |  |  | 4 |  |  |  | 3 |  |  |
| 9 |  |  |  |  |  | 5 |  |  |  |  |  | 3 |  |  |

Fig. 21: Number of letters generated from the chords in each cycle, triple, quadruple, and first and second balf.

[^19]

Fig. 22: Number of letters from the chords dropped in each cycle, triple, quadruple, and first and second balf.

## 9. The Key Poem

In a footnote, Rosenthal wondered about the meaning of the frequently encountered formulation that letters produced were marked "(as belonging) to the verse of the poem" ${ }^{78}$ In places a number is specified that is employed for this purpose. In the second cycle, one generates a " w " and "marks it with four [?] (as belonging) to the verse of the poem". ${ }^{79}$ Some values are added to it, resulting in 17, which is converted back by looking it up in the chords, yielding "alif", as mentioned in the previous section (see Figure 20, second row, third to last sign). The key poem is employed as a fixed lookup table assigning each letter a number and vice versa, comparable to an abjad with a mapping different from the conventional one. The substitution alphabet can be used in both directions and represents the generative "motor" of the procedure. Characters produced are transformed back into different quantities, which in turn can be further processed. Converting 1 to "alif" and then "alif" back to 1 would, in contrast, only yield a monotonous sequence, IIIII ... The question remains as to why the "w" is annotated with 4 , when it is only the second letter in the key poem. Figure 23 presents all characters with their position in the verse and their respective "marks". Where the text does not mention a number, the cell in the table has been left blank.

[^20]| $\mathrm{t}^{1}$ | $\mathrm{w}^{1}$ | $\mathrm{w}^{2}$ | ,${ }^{\prime}$ | $\mathrm{q}^{1}$ | $\mathrm{~s}^{1}$ | $\mathrm{~b}^{1}$ | $\mathrm{z}^{1}$ | $\mathrm{r}^{1}$ | , | , | $\mathrm{r}^{2}$ | $\mathrm{~s}^{2}$ | ${ }^{\prime} 2$ | $\mathrm{t}^{2}$ | $\mathrm{q}^{2}$ | $\mathrm{~b}^{2}$ | ${ }^{4}$ | $\mathrm{r}^{4}$ | $\mathrm{q}^{4}$ | ${ }^{8}$ | r |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | 2 | 2 | 3 | 13 | 1 | 27 | 15 | 24 | 25 | 34 | 24 | 1 | 3 | 16 | 13 | 27 | 3 | 24 | 13 | 3 | 5 |
|  |  | $\mathbf{4}$ |  |  |  | $\mathbf{2 7}$ | $\mathbf{1 5}$ |  |  |  | $\mathbf{4 8}$ | $\mathbf{2}$ | $\mathbf{6}$ | $\mathbf{3 2}$ | $\mathbf{2 6}$ | $\mathbf{5 4}$ | $\mathbf{1 2}$ | $\mathbf{9 6}$ | $\mathbf{5 2}$ | $\mathbf{2 4}$ |  |


| ${ }^{16}$ | $\mathrm{r}^{1}$ | $\mathrm{~h}^{1}$ | $\mathrm{r}^{2}$ | $\mathrm{~h}^{2}$ | $\mathrm{l}^{1}$ | $\mathrm{~d}^{1}$ | $\mathrm{r}^{4}$ | $\mathrm{~s}^{4}$ | h | $\mathrm{l}^{2}$ | $\mathrm{~d}^{2}$ | $\mathrm{y}^{1}$ | $\mathrm{~s}^{8}$ | $\mathrm{r}^{4}$ | ${ }^{3} 2$ | m | $\mathrm{t}^{4}$ | , | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | 24 | 14 | 24 | 14 | 4 | 37 | 24 | 1 | 33 | 10 | 37 | 7 | 1 | 24 | 3 | 8 | 16 | 9 | 4 |
| $\mathbf{4 8}$ | $\mathbf{2 4}$ |  | $\mathbf{4 8}$ | $\mathbf{2 8}$ |  | $\mathbf{3 7}$ | $\mathbf{9 6}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{2 0}$ | $\mathbf{7 4}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9 6}$ | $\mathbf{9 6}$ |  | $\mathbf{6 4}$ | $\mathbf{( 9 )}$ |  |

Fig. 23: Letters generated with real position in verse and "marks" from key poem.

The index that has been added to the characters solves the riddle of their "marking as belonging to the verse". It is a multiplicand called the "letter cycle", which starts from 1 and is doubled every time the respective sign is looked up again. ${ }^{80}$ Symbols are "marked" with the product of their position in the key poem and this value. Several times in the procedure (framed in Figure 23) " $r$ " is converted. At the first instance, no number is mentioned, but the resulting quantity is doubled at the second place, quadruplicated at the third, and so on. The " $w$ " was translated to 4 because it had been looked up before, at the second position. One can only speculate that every character was annotated with a value, and that the text only mentions it explicitly when the number is employed subsequently. The scale is reset when the "mark" equals or is greater than 96 , as shown by a comment in the text when the first " r " in the lower part of the table is generated: "Having reached ninety-six, the whole thing starts from the beginning, which is twenty-four." ${ }^{81}$

This theory accounts for all quantities obtained with the help of the key poem. It only fails to explain why the " $h$ " marked in grey in the lower part translates to 5, even though its position is 33 (the text seems to be corrupted at this place). ${ }^{82}$ Additionally, the last " r " should have been converted to 24 , not 96 . Ibn Khaldūn commented: "Its sign is ninety-six, which is the end of the second cycle of letter cycles", but no remark is provided at the previous instance of the character, where it was already translated to 96 and should have been reset. ${ }^{83}$ Of the "alifs"

[^21]highlighted, the second and third follow a special procedure and are looked up at a later position in the poem: this is why the value is only doubled on the fourth occasion.


Fig. 24: The list at the conclusion of the passage.
At the end of the section on the $z \vec{a} i r i j a$, the letters of the verse are listed under each other with numbers written on their left (fig. 24). ${ }^{84}$ Next to them, the twelve cycles with their respective start values are noted, which creates the misleading impression that they would somehow be guided by the characters of the poem. In fact, it was only written down in this way to facilitate its main function, the conversion of letters to numbers, and the proximity occurred accidentally. The term "key poem" should also be understood in the cryptologic sense of the

84 Muqaddimah, trans. Rosenthal, vol. 3, p. 212 [III, 177].
term. It is not surprising that the end of the list was treated rather carelessly, as Rosenthal noted, since only the first appearance of a letter serves the conversion to numerals. Of the four characters "written to one side", three already occurred earlier in the verse, and none of them are accessed in the course of the procedure. ${ }^{85}$
In Figure 25, positions that are used to generate numerals as "belonging to the poem" have been marked with indexes showing the order of events, as before. Less frequently, the verse is employed in the opposite direction, to convert quantities to letters. The respective locations have been circled and the superscript underlined. The characters are generally counted from the beginning and left in place afterwards. On two occasions the calculation continues from the last position looked up, in which case a "c" was added to the index.


Fig. 25: Key poem with bi-directional operations marked.

The quantity of characters produced through the verse rises regularly by one in each triple, as can be seen in Figure 26.

| I | II | III | IV | V | VI | VII | VIII | IX | X | XI | XII | R1 | R2 | R3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | q | r, ${ }^{\text {, }}$ |  |  | () $)^{86}$ | ', h | h | , | s, 1 | r | h | t |  |
| 1 |  |  | 2 |  |  | 3 |  |  | 4 |  |  | 2 |  |  |
| 3 |  |  |  | 2 |  |  |  | 5 |  |  |  | 2 |  |  |
| 3 |  |  |  |  |  | 7 |  |  |  |  |  | 2 |  |  |

Fig. 26: Number of letters generated from key poem in each cycle, triple, quadruple, and first and second balf.

85 Muqaddimah, trans. Rosenthal, vol. 3, p. 213, fn. 916.

## 10. The Reverse Table

The lookup of values on the reverse table, the largest and most complicated artefact employed ( $129 \times 55=7095$ fields with several thousand characters), is very difficult to reconstruct. At the beginning of each cycle, its basic number is entered at the side of eight, starting from the position in the last and counting upwards, and then all operations are carried out in the row reached. A detail permits reconstruction of the vertical movement with relatively high certainty. In the tenth cycle, the text reads: "One goes up nine on the side of eight. There is an empty (field). One goes up another nine and gets into the seventh (field) from the beginning. ${ }^{" 87}$ Adding up the preceding vertical steps, one gets: $8+5+1+9$ $+5+1+9+5+1=$ row 44 (see Figure 10). Going up nine from there, the pointer is located in line 53 , counted from the bottom. Even if only three signs are found here, it is not completely clear why this row is considered empty. If one climbs up another nine, the pointer goes over the top of the table and, restarting to count at the bottom, stops in the seventh line from the beginning, in agreement with the text, $53+9=62,62 \bmod 55($ the total number of rows $)=7$.

| I | II | III | IV | V | VI | VII | VIII | IX | X | XI | XII | R1 | R2 | R3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| t | n, w | z | ', | b | , | q | , , r | r | d, r |  | d, y |  | , 1 |  |
| 4 |  |  | 4 |  |  | 4 |  |  | 4 |  |  | 2 |  |  |
| 6 |  |  |  | 5 |  |  |  | 5 |  |  |  | 2 |  |  |
| 8 |  |  |  |  |  | 8 |  |  |  |  |  | 2 |  |  |
| 18 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Fig. 27: Number of letters generated from reverse table in each cycle, triple, quadruple, and first and second balf.

Figure 27 presents a count of the characters taken from the back of the artefact. Four of them are produced in each triple in a perfectly regular way. However, considerable difficulties were encountered in establishing the horizontal

[^22]movement and precisely locating the signs mentioned. Values are entered in the "front", and then either the symbol upon which the number "falls" is taken, or the "surface" is consulted and the letter obtained from there. The following tentative remarks are written in the hope of providing some more "shaky ground" on which to base future decryption attempts. The most successful strategy was to count only cells filled with characters in the current row, omitting the field of the "side of eight" and each time restarting from the right side, unless explicitly indicated otherwise in the text.

In Figure 28 the procedure in the reverse table has been noted. The first column contains the number of the cycle, the second the vertical movement in the "side of eight", and the third, the horizontal movement on the "front". In the fourth column, the resulting location of the pointer is found, following the numbering introduced in Figure 10 and specifying first the x , and then the y position. The fifth column gives the correction that had to be applied in some cases to reach the character mentioned in the text. If the sign was taken from the current row of the table, it was entered in the sixth column, and when the "surface" was looked up, it was entered in the seventh. If both signs were mentioned, the one not employed in subsequent operations has been put in parentheses. Next follows the numeral value of the sign and any calculations applied before converting it to the letter that results, which is found in the last column.

To illustrate the notation used, the second cycle will be discussed as an example. At its beginning, the pointer at the side of eight, which is located in the eighth row, moves up 5 , and consequently reaches field [1/13] (see Figure 10). On the front, one enters 17 and then 5 , finding the zimām symbol $\varphi$ at [42/13]. Its value is 500 , but it is scaled into the tens, 50 , which corresponds to " n " according to the abjad order. Then, 5 is looked up in the same row, leading to field [24/13], which contains $\mathbf{0}, 5$. Turning now to the surface, that is, the top of the column, one finds $I, 1$. The values obtained are added, $5+1=6$, and the result is converted to the letter " $w$ ".

Most of the signs required for the procedure can be found in the exact field that is reached. Operations that work as described in the text have been marked in grey in Figure 28. If a certain fuzziness is accepted, some more symbols can be located in the vicinity to the right or left of the correct spot. Distortion of values is all the more to be expected, since two complete columns are apparently missing from the table, which was said by Ibn Khaldūn to measure 131 squares horizontally.

It can be seen that the relatively high success rate at the beginning decreases after the tenth cycle. Possibly, from this point on the vertical movement was executed differently than assumed. Two "fixpoints" may help to establish the

| cycle | ymov | xmov | location | corr： | table | surface | numeral | letter |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I． | 8 |  | 1／8 |  | 8 |  |  |  |
|  |  | $15(3 \times 5)$ | 17／8 |  | （8） | 1 | 3 |  |
|  |  | 9c | 26／8 |  | $\downarrow$ |  | 400 | t |
| II． | 5c |  | 1／13 |  | 8 |  |  |  |
|  |  | $17+5$ | 42／13 |  | 9 |  | $500=50$ | n |
|  |  | 5 | 24／13 |  | － | 1 | $5+1=6$ | w |
| III． | 1c |  | 1／14 |  | 8 |  |  |  |
|  |  | 13 | 34／14 | （＋1） |  | $\stackrel{\square}{-}$ | $3 \times 2+1=7$ | z |
| IV． | 9c |  | 1／23 |  | 8 |  |  |  |
|  |  | 9 | 24／23 |  |  | 1 | 1 | ， |
|  |  | 11 | 27／23 ${ }^{88}$ |  |  | 1 | 1 | ， |
| V． | 5c |  | 1／28 |  | 8 |  |  |  |
|  |  | 26 | 50／28 |  | ＜ |  | 2 | b |
| VI． | 1c |  | 1／29 |  | 8 |  |  |  |
|  |  | 18 | 36／29 | （＋1） |  | 1 | 1 | ， |
| VII． | 9c |  | 1／38 |  | 8 |  |  |  |
|  |  | 10 | 35／38 | （－2） | 9 |  | $\begin{aligned} & 500=50, \\ & 50 \times 2=100 \end{aligned}$ | q |
| VIII． | 5c |  | 1／43 |  |  |  |  |  |
|  |  | 5 | 20／43 |  |  | 1 | 1 | ， |
|  |  | 52 | 101／43 | （－3） | ＜ |  | $2=200$ | r |
| IX． | 1c |  | 1／44 |  | 8 |  |  |  |
|  |  | 52 | 101／44 | （－2） | ＜ |  | $2=200$ | r |
| X． | 9c |  | 1／53 |  | \％ |  |  |  |
|  | 9c |  | 1／7 |  | 8 |  |  |  |
|  |  | 36 | 92／7 |  | 1 |  | 4 | d |
| （ |  | 9 | $17 / 7$ |  | 8 |  | $8 / 2=4$ | ） |
| （ |  | 18 | 27／7 |  | J |  | $\begin{aligned} & 1=10 ; \\ & (10-2) / 2=4 \end{aligned}$ | ） |
| （ |  | 27 | 40／7？ |  | c |  | $(10-2) / 2=4$ | $)^{89}$ |
|  |  | 26 | ？ |  | ヶて， |  | 200 | r |
| XI． | 5c |  | 1／12 |  | 8 |  |  |  |
|  |  | 5 （4？） | $5 / 12^{90}$ |  | \％ | 1 | 1 |  |
| XII． | 1c |  | 1／13 |  | 8 |  |  |  |
|  |  | 1 | 1／13 |  | 8 |  | $8 / 2=4$ | d |
|  |  |  | ？ |  |  | 5 | $5+5=10$ | y |
| R1 | 9c |  | 1／22 |  |  |  |  |  |
|  |  | 9 | ？ |  | （て） | $\stackrel{\square}{-}$ | $3 \times 9+7-1=33$ |  |
|  |  | 18 | ？ |  |  | 1 | 1 |  |
| R2 | 5c |  | 1／27 |  |  |  |  |  |
|  |  | 9 （39？） | ？ |  | （ $\boldsymbol{\gamma}$ ） | 1 | 1 | ， |
|  |  | 9 | ？ |  | $\gamma$ |  | 30 | 1 |

Fig．28：Lookup process in reverse table．
location of the pointer towards the end. In the first "result", R1, the number 9 falls on ghubār 2, $\mathbf{Z}$, in the table, while the surface contains 4 , $\dot{\sim}$. Counting 9 in the corresponding row leaves the pointer in [38/22]. In the vicinity of this address, the coincidence of the two signs only happens in fields [46/27], [50/28] and [34/29]. In the second result (R2), the combination of ghubār 30, $\boldsymbol{\gamma}$ in the table and "alif" at the top, which should be located at [26/27], can be found at [24/28] and [23/28] to [23/31]. The author has been unable to determine a coherent procedure that would connect these coincidences.

## 11. The Complete Routine

The question has been raised how to best describe algorithms, yielding impressively diverse answers and going as far as proposing to rely on Critical Theory to perform the task. ${ }^{11}$ The position of this chapter is simple: the best representation and basis for investigation is the most concise and least ambiguous one, which approximates closest to its complexity.

Figure 29 condenses the operation in its entirety into a sort of scripting language. In the first row, the cycle together with its number is found, in the second, the steps taken are encoded according to the legend, and in the third are the resulting letters. Arithmetic operations are noted without precedence, just in the sequence they are performed in. The procedure in the different cycles follows no discernable regularity concerning the order in which the different parts of the system are employed. In the first, the table is looked up, then the chords (T-C); in the second, table, surface, and then chords ( $\mathrm{T}-\mathrm{S}-\mathrm{C}$ ); in the third, poem, chords, chords, surface ( $\mathrm{P}-\mathrm{C}-\mathrm{C}-\mathrm{S}$ ). No description of how the sequence of events is determined is found in the text. Therefore, it cannot be established whether it is simply irregular, or because of some rule it is conditionally dependent on certain previous results.

In the last step of the procedure, the symbols are shuffled by dividing the sequence into two parts, one of 24 and the second of 23 characters, between the "alif" and " $r$ " in the eighth cycle, and then writing down the first letters of both,

[^23]| init |
| :--- |
| zodiac: 4 , degree: $1, \mathrm{v} 1: 8, \mathrm{v} 2: 5$ |


| $\mathrm{I}(\underline{9})$ | II $(\underline{17}, \underline{5})$ | III $(\underline{13}, \underline{1})$ |
| :--- | :--- | :--- |
| $9 \mathbf{T m}, 9+4,13 \mathbf{C m}$ | $17+5,22 \mathbf{T s}, 5 \mathbf{T}+\mathbf{S m} 4,4+8+5,17 \mathbf{C m}, \mathrm{Cd} 3$ | $13 \mathbf{P m}, 13 \mathbf{C m}, 1 \mathrm{c} \mathbf{C m} 27,13 \mathbf{S} \times 2+1 \mathrm{~m} 15$ |
| $\mathrm{t}, \quad \mathrm{w}$ | n, w, | q, s, b, $\quad$ z |


| IV (9) | $\mathrm{V}(17, \underline{5})$ | VI $(13, \underline{1})$ |
| :---: | :---: | :---: |
| Cd2, 9cPm, 9Sm, 9cPm, Cd1 , 9 $\times 2,18 \mathrm{Cm} 48,18 \mathrm{Cm} 2,9+2,11 \mathrm{Sm} 6$ | Cd2, 5×2+17, 27Cm32, 17-2, 15Cm26, 26Tm54 | Cd2, 1+4+13, 18Sm12, Cd2, 18+2, 20Cm96 |
| r , , r | t, q, b | , r |


| VII (9) | VIII (17, $\underline{\text { ) }}$ | IX (13, 1) |
| :---: | :---: | :---: |
| Cd2, 9+1, 10Ts $\times 2 \mathrm{~m}$ 52, $52-2-9,41 \mathrm{Cm} 24,41 \mathrm{P}$ | Cd2, 5Pm, 5 Sm48, 48-1+5, 52Tsm24, 24+5-1/2, 14Pm | $13 \times 4,52 \mathrm{Tsm} 48,13+1,14 \mathrm{Pm} 28,14-7, \mathrm{Cd} 2,7 \mathbf{C m}$ |
| $\mathrm{q}, \quad$ ', (') | ', ', r, h | r, $\quad \mathrm{l}, \mathrm{l}$ |



| init |
| :--- |
| $9 \underline{3}$ letters, $93: 12=\underline{7}, 93 \% 12=\underline{9}$ |


| R1 (9) | R2 (17, 5) | R3 (13, 1) |
| :---: | :---: | :---: |
| Cd2, $9 \times 3+1,28 \mathbf{C m 9 6}, 9 \mathbf{S} 3,3 \times 9+7-1,33 \mathbf{P}, 9 \times 2,18 \mathbf{S} 1,1 \mathbf{C m}$ | Cd2, $5 \times 3+1,16 \mathbf{P m} 64,5+3+1,9 \mathrm{~T} 30 \mathrm{~S}(\mathrm{~m} 9), 9 \mathrm{Tm}$ | $13+3+1+1,18 \mathrm{C}$ |
| , $\mathrm{h}, \mathrm{m}$ | t, $\quad 1$ | 1 |



Fig. 29: Overview of the complete zā’irja procedure.
after that, the second ones, and so on. This basic columnar transposition rearranges the output to: "t, r, w, h, n $|\mathrm{r}, \mathrm{w}, \mathrm{h}|$ ', $\mathrm{l}, \mathrm{q}, \mathrm{d}, \mathrm{s} \mid$ ', $\mathrm{b}, \mathrm{r}, \mathrm{z} \mid \mathrm{s}, \mathrm{r}, \mathrm{h}$, ' $\mid$
 pretation as an Arabic verse, "Tarūhanna rūhu l-qudsī ubriza sirruhā / Li-Idrīsa fa-starqā bi-hā murtaqqā l-‘ulā" - "The Holy Spirit will depart, its secret having been brought forth / To Idrīs, and through it, he ascended the highest summit". As the shuffling makes it difficult to foresee the final location of the letters, it is hardly imaginable that the practitioner composed verses at his will and only simulated the formal procedure by picking calculations that result in the necessary numbers.

## 12. The Transfer of Forms

In the course of the operations, values obtained are often added, as in the second part, where the " $w$ " is marked with four "as belonging to the poem", and this is then summed up with eight, the first value of the moment, and five, the remainder of the cycle's number, resulting in 17 , which is looked up in the chords and generates "alif". ${ }^{92}$ Less frequently, subtraction is employed, as in the seventh portion of the routine, where " q " is converted to 52 in the verse, of which two is dropped, without explanation, together with the nine that is the remainder of this section's principal value, resulting in 41 , which is entered in the chords and yields "alif". The " q " itself was the outcome of another operation, that of scaling between units, tens and hundreds. One "enters ten on the front (recto) of the table, and gets thus to a stop at five hundred. It is, however, (counted) only as fifty, $n$." The text does not indicate how the scale is determined. The final letter is reached by yet another technique, the frequent doubling or halving of values: "It is to be doubled. Thus, it is $\mathbf{q}$ ", 100 in abjad. ${ }^{93}$ The most unusual operation occurs in the calculation with the uss (literally, "base"). While the term usually means the exponent of a term, in this context it seems to indicate the unit fraction of a composite number. In the fifth part, " t " is marked with 32 , and then the practitioner "subtracts the two which is at the base of thirty-two, from seventeen", the cycle's principal value, and enters fifteen into the chords, generating " q ". ${ }^{94}$ Where the uss is taken, the unit number has been underlined in Figure 29, like all other output that is employed subsequently. At places, a value of one is added or subtracted in the course of the routine, in cycles $7,8,11$ and 12 , at the end of the second and third quadruple, and in all three results.

The arithmetical procedures on the $z \bar{a} \bar{a}^{\prime} i r j a$ relate to the abacus. As shown above, the modulo operation, which is called taksir ("breaking down") in Arabic letter science, occurs when quantities have to be carried over on the calculation device. ${ }^{95}$ In the instrument of divination, it is found on the front table, where the selection process cycles depending on the ascendant. The numbers of the principal

[^24]parts are calculated employing modulo arithmetic and the consultation of the rectangular table and key poem restarts at the beginning when the end is reached (cf. the beginning of Section 10). The scaling of quantities, one of the main properties of the abacus and closely related to the system of place value, also occurs on the $z \vec{a} \dot{i} i j a$, as mentioned above. Equally, the retention of the uss is only possible if numbers are dissected into ones, tens, hundreds, and so on. Humboldt wrote: "In the Orient, the negromantic [sic] art of sand is called raml. Continuous or broken lines and points, which represent the elements, guide the soothsayer. [...] [T]he parallel lines of the magic books, similar to notes, knotty, often broken, seem to be mere graphical projections of these chords of calculation and thought [which make up the abacus]."96
A number of similarities support the opinion that Llull constructed his paper machine inspired by the algorithmic artefact of the Muslims. Charles Lohr wrote the $z \bar{a}$ 'irja provided "many close parallels not only to the association of letters with the elemental powers in Lull's early Artes and to the application of them to astrology in the Tractatus novus de astronomia, but also to the triangles and revolving circles which have been responsible for much of the misunderstanding of Lull's intention in the Art". ${ }^{97}$

- The front table of the $z \bar{a} i r i j a$ and the Majorcan device are visually similar. In the divinatory artefact, a major part of the answer is determined by a cyclical revolution, the natural movement of the stars as seen from Earth, which also determines time. In Llull's Art, turning the disks in regard to each other effects the recombination of terms, an element that seems absent from the Arabic device. But here also, some of the letters taken off the chords are selected, and then shuffled with characters that originate from other sources.
- The $z \bar{a}{ }^{\prime} i r j a$ is made of one disk revolving in regard to a natural fixpoint, the ascendant, which is conventionally represented at the nine o' clock position. The Majorcan philosopher replaces the outer reference by a duplication of the artefact, which now only correlates to itself.

96 Humboldt, Systeme von Zahlzeichen, p. 216: "Im Orient wird raml die negromantische Kunst des Sandes genannt. Ganze oder gebrochene Linien und Puncte, welche die Elemente vorstellen, leiten den Weissager. [...] [D]ie notenartigen, knotigen, oft gebrochenen Parallellinien der Zauberbücher [...] scheinen nur graphische Projektionen von diesen Rechen- und Denkschnüren [, die den Abacus ausmachen.]." The German word "negromantisch" is not found in any of the relevant dictionaries and probably constitutes a misprint; translation from the German, and emphasis added by D.L.
97 Lohr, Christianus arabicus, p. 64. Lohr mainly refers to the Ars compendiosa inveniendi veritatem (1273/75), the Ars demonstrativa (1275/81), the Ars inventiva (1289/90) and the Ars generalis ultima (1308).

- On the $z \bar{a} \neq i r j a$, truth is generated dynamically, by processing single signs. The same is true of the Majorcan device, which recombines letters in a much more simple and straightforward way. Here, the algorithm does not need to perform the seemingly impossible, to generate words, because the symbols are already thought to represent concepts.
- While the letters B to K indicate varying contents in the Art, like the qualities of God or the categories of thinking, the signs are continuously converted between the alphabets on the $z \bar{a} ’ i r j a$. The same symbol may indicate a letter or a numeral and be translated between the two in different ways, depending upon which part of the system is employed.
The fact that only consonants are written down in Semitic languages permits the meaningful interpretation of many random permutations of symbols, which seems rather incredible seen from the point of view of the vowel alphabet. The German linguist Johannes Lohmann compared the differentiation of Indo-European and Semitic to the separation of science (epistème) and poetry (poìesis) in Ancient Greece; the "thinking" and the "acting" component of sign usage. "In the Semitic word form", he wrote, "the semantic act is part of the living language". ${ }^{98}$ This "magic" state of writing, which may have fostered the view that it was Godgiven, was illustrated in a more secular form by Claude Shannon in 1948: "The redundancy of a language is related to the existence of crossword puzzles. If the redundancy is zero, any sequence of letters is a reasonable text in the language and any two-dimensional array of letters forms a crossword puzzle." ${ }^{" 9}$

When Llull imported the $z \bar{a}$ 'irja technology into the realm of the vowel alphabet, it lost its function altogether because of the higher redundancy of this type of writing system, in which many possible letter combinations are meaningless. In a notation in which many shufflings of the signs produce a meaningful word, their free permutation arises quite naturally and playfully as a symbolic modus operandi. ${ }^{100}$

[^25]
## 13. Scrambling T-R-U-T-H

More than 600 years ago, the z $\bar{a} \backslash i r j a$ already provided a functionality which the constructors of search engines, and many others, are longing for today. ${ }^{101}$ Knowledge of the technique appears to have been forgotten, or may have been applied to other fields, like in Brecht's parable on the obsolescence and the cyclical return of artefacts from his radio theory, which Siegfried Zielinski quoted in the preface to the last Variantology volume. ${ }^{102}$ The man in this story at least knows what has fallen into oblivion. The archaeology of algorithms and their artefacts tries to prevent a state of affairs where suddenly certain things have never existed, which occurs after an alarmingly short time in the modern era.
The aims of artificial intelligence and 'ilm al-burūf (the science of letters) coincide: to provide a truthful answer to any question posed. But the knowledge of all facts of all times represents a tragic and highly paradoxical situation. It excludes human freedom, because it is based on the assumption that the fate of everything has already been written down. Even though the actor knows beforehand what will happen to him in the rest of his life, he is unable to change it, not even the slightest gesture, and more cruel than in "Groundhog Day": a man suddenly exists in a different time system, in which the same day is replayed in all eternity. ${ }^{103}$ Paradoxically, he is the only person capable of acting differently. The irony of the film is that, whatever he does, the end result will be the same. He will wake up the next morning to the same old day replayed. If, on the other hand, the question whether the destined future can be changed is affirmed, the Book of Truth must have been outdated when it was consulted. ${ }^{104}$ In this case, it would not contain fate since the beginning, but only the calculated results of all prior action, like Laplace's "demon":
"We ought then to consider the present state of the universe as the effect of its

[^26]previous state and as the cause of that which is to follow. An intellect that, at a given instant, could comprehend all the forces by which nature is animated and the respective situation of the beings that make it up, if moreover it were vast enough to submit these data to analysis, would encompass in the same formula the movements of the greatest bodies of the universe and those of the lightest atoms. For such an intelligence nothing would be uncertain, and the future, like the past, would be open to its eyes." ${ }^{105}$

If the truth-telling device is constructed algorithmically, somehow with all events up to this point at its disposal, the paradox is seemingly resolved. In fact it suffices to pose hypothetical questions to it to re-introduce the contradiction, having decided beforehand to undertake every measure to escape the predicted fate: "Will the stone reach the floor if I drop it?". ${ }^{106}$ It is easier to imagine that by some mysterious power the oracle created some anomalies in reality than that it literally forced the hand of the human subject.

The $z \bar{a}$ irijab is based upon a similar assumption. 'Ilm al-burū$f$, the science of letters, belongs to the broader discipline of simiyā ("letter magic"). ${ }^{107}$ The term is usually derived from Greek $\sigma \eta \mu \varepsilon ̃ o v ~(" s i g n " ~ o r ~ " s i g n a l "), ~ t h e ~ w o r d s ~ " s e m a n t i c s " ~$ and "semiology" stem from. However, an Arabic dictionary, the Mubìt al-Mubīt of 1870, suggests the etymology derives from Hebrew שם י"shem yah" ("name of God"), and the names of God "certainly play a large part in sīmiyā". ${ }^{108}$ These practices appear to be based on the idea that all things were generated by permuting those 99 sign sequences, and that all future events are determined by their further unfolding, which proceeds perfectly regularly and according to laws. If the rules that guide this symbolical development can be determined, the course of the Real can be predicted and, consequently, the Truth revealed.

The fact that the preceding explanation of the $z \bar{a} \backslash i r j a$ operation has taken up over twenty pages and the length of the condensed expression in Figure 29 allow to estimate its high complexity. The Russian mathematician Andrey Kolmogorov described the concept as follows:

[^27]"One often has to work with very long sequences of symbols. Some of them, for example, sequences of digits in a five-place table of logarithms, admit a simple logical definition and can correspondingly be obtained by computation [...] according to a simple program. Others, however, presumably do not admit any sufficiently simple 'regular' construction: for example, a sufficiently long segment of a 'table of random numbers'." 109
In its high complexity, the procedure resembles an encryption algorithm. Letters are converted to numbers, looked up in different substitution alphabets, and finally transposed. At the time of Ibn Khaldūn's initiation (1370), cryptography was already advanced among the Arabs, as described in detail by Ibrahim AlKadi. Among the reasons for this that Al-Kadi furnishes is the translation of treatises from dead languages, some of which, dealing with alchemy, were enciphered, linguistic studies of Arabic, and the need to protect sensitive information in the administration of the state. ${ }^{110}$ In the thirteenth century, the scholar Ibrāhim Ibn Mohammad Ibn Dunainīr (1187-1229) devised an arithmetical cipher whose procedures are remarkably close to those of the $z \bar{a}$ 'irja. First, the single letters of the cleartext are converted to numerals according to the abjad system, and then they are decomposed into two summands, which are translated back into alphabetic signs. In this way, "h" ( $\tau$ ) is transformed to 8 , and then to $2+6$, resulting in "bo", بو, or to $1+7$, "az" (از). ${ }^{111}$ Alternatively, the value is simply doubled, 16, and expressed as a composite abjad numeral, $10+6$, , which can be read as "yo", or tripled, 24,

In 1467, the Italian polymath Leon Battista Alberti utilised the concentric circles that Llull had used to meditate and prove the truths of Christianity, that is, still in a revelational manner, to perform the opposite task: the concealment of communication. Previously, the outcome of wars had been predicted by scrambling letters on the $z \bar{a} ’ i r j a$; now messages containing the future of the battle, in the form of commands to be executed by the units in the field, were made

109 Andrey N. Kolmogorov, Report on 24 April 1963 to the Probability Theory Section of the Moscow Mathematical Society, quoted in: A.N. Shiryaev, Andreǐ Nikolaevich Kolmogorov. A biographical sketch of his life and creative paths, in: Kolmogorov in Perspective (Providence, RI, 2000), pp. 1-89, p. 64; cf. A.N. Kolmogorov, Three approaches to the quantitative definition of information. Problems of Information Transmission 1 [1] (1965): 1-7.
110 Ibrahim A. Al-Kadi, Origins of cryptology: The Arab contributions. Cryptologia 16 [2] (1992): 97-126, esp. 98-101. On p. 103, he lists Arabic cryptologists and their publications from the eighth to the fifteenth century.
111 The letter "w" also indicates the vowel "o" or "u" in Arabic.
112 Al-Kadi, Origins of cryptology, pp. 115 and 119, Fig. 7.
unreadable for the opponent by employing the same techniques. The aim was to remove the regularities of language, which would have permitted to read the strategic communication, and no longer to imitate the semantic laws of Nature's unfolding to predict events to come. However, to be decodable by the receiver, this operation as well needed to follow strict rules, whose traces also had to be concealed. This paradoxical situation led to the sensational break-in of a young Polish mathematician into the first machine of this kind employed on a broad scale, the German Enigma, in 1933. ${ }^{113}$


Fig. 30: The career of a technical form: Alphabetic disk of Alberti, 1467, and letter wheels within Enigma. ${ }^{114}$

[^28]
## 14. The $z \bar{a}$ 'irja's Effectiveness

The question whether the $z \bar{a} ’ i r j a$ was able to fulfil its function leads into the field of the Undecidable. To predict the behaviour and output of an algorithm in a formal way, for example, if it stops at all or not, has been demonstrated to be impossible in many cases. ${ }^{115}$ What can be said is that the complexity of the routines employed, substitution, transposition, and some simple arithmetical operations, are alone not sufficient to generate intelligible language, let alone regular rhythm and rhyme. All depends on the quality of the basis of the procedure, the seemingly arbitrary, rule-less two tables and the poem. If they were constructed following a simple regularity, it could be proven that the device cannot perform the claimed function. The late remnant of the $z \vec{a}$ ’irjab procedure that the British Orientalist Edward W. Lane (1801-1876) found in Egypt and published in 1836 depends entirely on the craftsmanship of the lookup table. ${ }^{116}$ Starting from a letter that he selected blindly, the soothsayer picked every fifth, resulting in one of the hardcoded recommendations, like "Do it without fear of ill". The operation is similar to the final columnar transposition executed on the $z \vec{a}$ 'irja, and the device is still ascribed to Idrīs.


Fig. 31: Late Egyptian zā’irja.

[^29]In the procedure on the original artefact, the question posed only plays a role in the form of the number of its letters, and is employed towards the end, in the so-called "results", when most of the signs, apart from the last seven, have been determined. By contrast two values are devoted to the world, the objective stellar situation. At the beginning of the passage, Ibn Khaldūn wrote:
"A question may have three hundred and sixty answers, according to the degrees (of the firmament). The answers to one question under a given ascendant differ in accordance with different questions (forming part of the question asked), which are referred to the letters of the chords of the $z \vec{a} \overrightarrow{i r j a}$ and (in accordance with) the operation applied to finding out the letters from the verse of the poem."

The signs of the question are added to the ones from the chords, and their number counted. The result has to lie between 88 and 96 , otherwise the text entered needs to be shortened. ${ }^{117}$ Consequently, the input to the system consists in 30 (degrees) $\times 12$ (possible rulers) $\times 9$ (different numbers of letters) $=3240$ discernable "situations" or possibilities to which it generates an answer. It is difficult to say if the routine further depended on the question. An enigmatic remark in the sixth cycle, after the first half of it has been executed, seems to imply that:
"At this point, one looks at the letters of the question. The (letters) that have come out (in the preceding operation) are paired with the verse of the poem, beginning at the end. One marks them with the letters of the question, so that it enters numerically into the verse of the poem. The same is done with every letter that comes out hereafter, in correspondence with the letters of the question. All letters coming out are paired with the verse of the poem, beginning at the end, and a mark is put on them."

If the only data that enters the routine is the number of signs, it is possible that each time it was consulted the astonishingly complex procedure only produced one of approximately 3,000 answers, which were in some way hardcoded in the tables and the poem. If no mechanisms exist that change the flow of the algorithm, it always generates the same text for a certain set of input values. In this case it would be functionally identical to its late Egyptian successor, only much larger. It is difficult to believe that such vast complexity would be employed merely to select one out of a set of different answers. A promising line of enquiry might be a frequency count of the tables, to find out how close they are to the normal distribution of letters in Arabic, as well as further cryptanalytic

117 Muqaddimah, trans. Rosenthal, vol. 3, p. 196f. [III, 162].
processing. ${ }^{118}$ Another possibility would be to question the device directly, for example, by asking:
"Does the zā̀irja show the truth?"
The artefact could be regarded as a very early experiment in the free algorithmic processing and conversion of symbols. The operations are executed like calculations, but follow rules different from the mathematical ones, being derived from another kind of truth. Signs are freely transformed, guided by signs. If Ibn Khaldūn was really initiated into the technique in around one hour, the level of penetration his text shows is astonishing. However, it seems that some of the information escaped him. It may be possible to further reconstruct the complete routine using the numerous other manuscripts on the subject written by authors less prominent today. It is hoped that the foregoing detailed analysis provides a more solid fundament for such an undertaking. "And God knows better." ${ }^{119}$

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[^0]:    1 Quoted by Henri Corbin, Histoire de la Philosophie Islamique (Paris, 1964), p. 205: "Sache que les secrets de Dieu et les objects de Sa science, les réalités subtiles et les réalités denses, les choses d'en haut et les choses d'en bas, sont de deux catégories: il y a les nombres et il y a les lettres. Les secrets des lettres sont dans les nombres, et les épiphanies des nombres sont dans les lettres. Les nombres sont les réalités d'en haut, appartenant aux entités spirituelles. Les lettres appartiennent au cercle des réalités matérielles et du devenir." Translation from the French, D.L.

[^1]:    2 Amador Vega, Ramon Llull and the Secret of Life [2002] (New York, 2003), p. 57, Fig. 1 p. 64.
    3 Cf. Charles Lohr, Christianus arabicus, cuius nomen Raimundus Lullus. Freiburger Zeitschrift für Pbilosophie und Theologie 31 [1-2] (1984): 57-88; Dominique Urvoy, La place de Ramon Llull dans la pensée Arabe. Catalan Review. International Journal of Catalan Culture 4 [1-2] (1990): 201-220; D. Urvoy, Penser l'slam. Les présupposés Islamiques de l'"art" de Llull (Paris, 1980); Armand Llinares, References et influences Arabes dans le Libre de contemplacio. Estudios Lulianos 24 [71] (1980): 109-127. The transliteration of the name varies considerably, "zā’irjah", "zā’irajah", "zā'irdja", "zā’iradja", "zāyirǧa" being but a few examples. The Arabic letters in زايرجة are, from right to left: ز- zāy, l - '(alif),
     a mixture of the Persian words for "horoscope, astronomical table" (zā’icha) and "circle" (dā’ira); cf. Ibn Khaldūn, The Muqaddimah. An Introduction to History, trans. Franz Rosenthal, 3 vols. (New York, 1958), vol. 1, p. 238 f. , fn. 364.

    4 Muqaddimah, trans. Rosenthal, vol. 1, p. 239 [I, 213]. (The page numbers in square brackets refer to the Arabic edition of Quatremère.) The many surviving Arabic manuscripts by various authors attest to the popularity the device once enjoyed; cf. the catalogue of the Princeton Collection of Islamic Manuscripts (New Series and Garrett Collection, Yahuda Series), which lists over 25 manuscripts on the topic; online: http://www.princeton.edu/~rbsc/department/manuscripts/islamic.html.

[^2]:    5"Muqaddima" signifies "Prolegomenon, introduction" in Arabic. The main printed editions of this work are, in chronological order: Ibn Khaldūn, Kitāb al-'Ibar wa-dīwān al-mūbtadā' wa-l-habar etc., ed. Naṣr al-Hūrīnī (Bulaq near Cairo, 1857, 1 vol.; reprint Bulaq, 1867/8, 7 vols.; in Arabic); Prolégomènes d'Ebn-Khaldoun, ed. Étienne M. Quatremère, 3 vols. (Paris, 1858; in Arabic); Prolégomènes Historiques d'Ibn Khaldoun, trans. William MacGuckin de Slane, 3 vols. (Paris, 1863-1868); Ibn Khaldūn, al-Muqaddimah, ed. 'Al̄̄ A. Wāfí, 4 vols. (Cairo, 1957-1962; in Arabic); Ibn Khaldūn, The Muqaddimah. An Introduction to History, trans. Franz Rosenthal, 3 vols. (New York, 1958); Ibn Khaldūn, Discours sur l'Histoire Universelle (al-Muqaddima), trans. Vincent Monteil, 3 vols. (Paris - Beyrouth, 1967/8); Ibn Khaldūn, Le Livre des Exemples, vol. 1: Autobiographie, Muqaddima, trans. Abdesselam Cheddadi (Paris, 2002).
    6 Muqaddimah, trans. Rosenthal, vol. 1, p. 239 [I, 213] and fn. 365. Cf. Carl Brockelmann, Geschichte der Arabischen Literatur, Suppl. vol. 1 (Leiden, 1937), p. 909.
    7 Muqaddimah, trans. Rosenthal, vol. 3, p. 196, fn. 880.
    8 Dates in this chapter generally follow the Christian system.
    9 Henri P. J. Renaud, Divination et histoire nord-africaine au temps d'Ibn Khaldun. Hespéris 30 (1943): 213-221, esp. 213-215. How the latter learned the fact from the former is unclear, since more than 600 years separate them.

[^3]:    18 Cf. Florian Cajori, A History of Mathematical Notation, vol. 1: Notations in Elementary Mathematics [1928] (New York, 1993), pp. 21-30; Georges S. Colin, Lorigine grecque des "chiffres de Fès" et de nos "chiffres Arabes". Fournal Asiatique 222 (1933): 193-215, esp. 193-203; cf. p. 202f., fn. 3: "It is odd to find that still at present, the majority of the functionaries of the Egyptian ministry of finances are Copts."
    19 A. Lazrek, Rumi Numeral System Symbols, 15 July 2006. http://www.ucam.ac.ma/fssm/rydarab/ doc/unicode/amos5l.pdf, p. 6.
    20 Cf. Mahdi Abdeljaouad, Le manuscrit mathématique de Jerba. Une pratique des symboles algébriques Maghrébins en pleine maturité, in: Actes du 7ième Colloque Maghrébin sur l'Histoire des Mathématiques Arabes, ed. Abdallah El Idrissi and Ezzaim Laabid (Marrakech, 2005), pp. 9-98. Online http://math.unipa.it/~grim/MahdiAbdjQuad11.pdf, p. 21.

[^4]:    23 Cf. Ifrah, Universal History, pp. 291-294.
    24 The origins of the system in India is supported by many documents, but is by no means undisputed; cf. Ifrah, Universal History, pp. 356-439. For a controversial position, see Solomon Gandz, The origin of the Ghubār numerals, or The Arabian abacus and the articuli. Isis 16 (1931): 393-424.
    25 Muqaddimah, trans. Rosenthal, vol. 3, p. 197, fn. 882. For the various forms of the numerals in the course of their history, see Ifrah, Universal History, pp. 534-539. According to Ifrah, "the oldest

[^5]:    rays, in: Variantology 2. On Deep Time Relations of Arts, Sciences and Technologies, ed. Siegfried Zielinski and D. Link (Cologne, 2006), pp. 15-42.
    32 Gandz, Origin of Ghubār, p. 395f.; cf. Ifrah, Universal History, p. 207.
    33 Muqaddimah, trans. Rosenthal, vol. 1, p. 239 [I, 214].
    34 Colin, Origine grecque, pp. 208-210. This appears less astonishing when one considers that the impractical Roman signs are still used today when calculations are not intended; for example, in the numbering of book volumes.
    35 Colin, Origine grecque, pp. 194-196, 210. The Moroccan lawyer Abdelhamid Benmakhlouf kindly informed me that today, the numbers are still used by the Hebrew notary's office in his country (pers. comm. 22 July 2008).

[^6]:    36 MS Ahmet III, 3042, Topkapi Saray library, Istanbul, vol. 1, fol. 237. The designations "front" and "back" derive from the counting method of folios, which differentiates between recto and verso.

[^7]:    40 Mālik b. Wuhayb lived from 453-525 A.H., the early eleventh century; cf. Muqaddimah, trans. Rosenthal, vol. 1, p. 240, fn. 372; Seyyed Hossein Nasr and Oliver Leaman, eds., History of Islamic Pbilosophy, Part I (London, 1996), p. 296f.: "Among the logicians of Andalusia was Mālik ibn Wuhayb, famed for his learning in many sciences, including astronomy and (judicial) astrology, and known in his day as the Philosopher of the West." He later "turned his talents to divinity" and gave up on "open discussions of philosophy [...] because of the attempts on his life".
    41 Muqaddimah, trans. Rosenthal, vol. 3, p. 211 [III, 176].
    42 "Tu possèdes la question de la grande forme naturelle. Conserve donc les doutes étranges qui ont été soulevés et que le zèle peut dissiper." Cheddadi, Livre des Exemples, p. 1008. Translation from the French, D.L.
    43 Cf. Mehr Afshan Farooqi, The secret of letters. Chronograms in Urdu literary culture. Edebiyāt 13 [2] (2003): 147-158; Ifrah, Universal History, pp. 250-252.
    44 There is a short summary in the first volume, Muqaddimah, trans. Rosenthal, vol. 1, pp. 238-245 [I, 213-220]. The extensive account is found in vol. 3, pp. 182-214 [III, 146-179].

[^8]:    45 Muqaddimab, trans. Rosenthal, vol. 3, p. 197, fn. 880. The $z \bar{a}$ ' $i r j a$ poem Rosenthal refers to, which is attributed to as-Sabtī, precedes the detailed discussion of the device in the third volume. The complete obscureness of this long passage is probably due to the employment of some esoteric or cryptographic sort of code.
    46 After Rosenthal's death, they were given to Sourasky Library, Tel Aviv University, and have not yet been classified and catalogued (Miri Lipstein, director of the library, personal communication, 4 May 2008).

    47 Muqaddima, trans. Monteil, vol. 3, p. 1123, fn. 1. Translation from the French, D.L. The text is in fact not confused; its complexity is confusing.
    48 Cf. fn. 87 below.
    49 Henri Veyrier, Encyclopédie de la Divination (Paris, 1982), p. 153. Translation from the French, D.L. In the cyclical career of artefacts described by Thompson, desuetude corresponds to the covert state of rubbishness; cf. Thompson, Rubbish Theory, p. 10. Without a concentrated attempt at its resurrection, an artefact, especially an algorithmic one, very rapidly becomes enigmatic, the end result being an "Odradek"; cf. Franz Kafka, The Cares of a Family Man, in: The Complete Stories (New York, 1971), p. 428; D. Link, Enigma Rebus. Prolegomena to an Archaeology of Algorithmic Artefacts (forthcoming).

[^9]:    50 http://www.el7akeem.com/vb/showthread.php?p=28766, a website registered in Egypt. Other domains the word appears on are located in Saudi Arabia (4), Egypt (3), Morocco (1), Lebanon (1), and Syria (1). The author is indebted to Hicham Kerrouri and Samir Awaragi for translating these and other passages from Arabic.
    51 Cf. Andrzej K. Konopka, Sequences and codes. Fundamentals of biomolecular cryptology, in: Biocomputing. Informatics and Genome Projects, ed. Douglas W. Smith (San Diego, 1994), pp. 119-174, p. 122f.: "Several methods for determining classification codes in large collections of nucleotide sequences are described in this survey. All these methods are analogous to techniques for [...] breaking monoalphabetic substitution cryptosystems [...]. In case of cryptology proper, the structure of 'functional messages' is known because the plain language is known. [...] The semantic aspects of 'functional messages' [in genomics], however, remain unknown even in principle because we do not know the 'language of Nature' nor is it obvious that such a language exists."

[^10]:    56 A list of the manuscripts Rosenthal consulted can be found in Muqaddimah, trans. Rosenthal, vol. 1, p. lxxxviii. Rosenthal wrote that an "exhaustive utilisation of all the manuscripts" could be expected from a forthcoming edition of the Muqaddimab by Muhammad Tawit at-Tanji, which unfortunately never appeared in print. However, the translation by Cheddadi mentioned in fn. 5 above is based on a large number of copies consulted.
    57 Lewis Carroll, Through the Looking Glass (London, 1871), p. 150f.
    58 Cf. Muqaddimah, trans. Rosenthal, vol. 2, p. 382 [II, 341f.]: "Arabic writing at the beginning of Islam was, therefore, not of the best quality nor of the greatest accuracy and excellence. It was not (even) of medium quality, because the Arabs possessed the savage desert attitude and were not familiar with crafts. [...] The men around Muhammad wrote the Qur'ân in their own script, which was not of a firmly established, good quality. Most of the letters were in contradiction to the orthography required by persons versed in the craft of writing."

[^11]:    59 Muqaddimah, trans. Rosenthal, vol. 1, p. 243f. [I, 217f.]. In 1684 the Archbishop of Canterbury, John Tillotson, gave the following explanation of the expression "hocus-pocus" which appears in the translation: "And in all probability those common juggling words of hocus pocus are nothing else but a corruption of hoc est corpus, by way of ridiculous imitation of the Priests of the Church of Rome in their trick of Transubstantiation." John Tillotson, A Discourse Against Transubstantiation (London, 1685³), p. 34.
    60 For the count of letters on the chords, Rosenthal's transliteration (Figure 8) was used.

[^12]:    61 Zimām and ghubār numbers were translated to letters and in this case noted in superscript.
    62 The matching process can be compared to "anagramming" in cryptology; cf. David Kahn, The Codebreakers. The Story of Secret Writing (New York, 1967), p. 103.
    63 Geoffrey Sampson, Writing Systems. A Linguistic Introduction (Stanford, 1990), p. 95f.
    64 Given a set of input symbols, a program produces a certain output. If both are identical, the algorithm performs nothing.

[^13]:    65 Papyrus was widely used in the Islamic world even after the Arabs had acquired the technique of paper production from the Chinese at the end of the eight century; Ifrah, Universal History, p. 532f., p. 516.

    66 In Bletchley Park jargon during the Second World War, the term indicated the reception of two messages that had been encoded with exactly the same key settings and thus permitted a break-in.

[^14]:    68 Abdeljaouad, Manuscrit mathématique, p. 6.

[^15]:    69 Cf. Moshe Hallamish, An Introduction to the Kabbalab (New York, 1999), pp. 43-46.
    70 Cf. Muqaddimah, trans. Rosenthal, vol. 1, p. 241 [I, 215]: "In the language (used here) the 'base' is

[^16]:    the sign's distance from the last rank, in contrast to the (meaning of) 'base' in the language of the astronomers [?], where it is the distance from the first rank." The Arabic word in question is uss.
    71 Muqaddimah, trans. Rosenthal, vol. 3, p. 199 [III, 163], and vol. 1, p. 242 [I, 216].
    72 Muqaddimah, trans. Rosenthal, vol. 3, p. 200 [III, 164].

[^17]:    73 Muqaddimah, trans. Rosenthal, vol. 3, p. 200 [III, 165], see also fn. 895.
    74 This could be the outcome of the ascendant being in the 30 th degree of the 5 th sign, if seven cycles are counted before one of the signs is hit, $(30+1) \times 5=155 ; 155 \bmod 12=11$.

[^18]:    75 Cf. the second problem in Jakob Bernoulli's Ars Conjectandi, J. Bernoulli, The Art of Conjecturing, together with Letter to a Friend on Sets in Court [1713], trans. Edith D. Sylla (Baltimore, 2006), pp. 181-188.

[^19]:    76 They are "dropped" only at one instance in Muqaddimah, trans. Rosenthal, vol. 3, p. 202 [III, 166], and "picked" on pp. 203-210 [III, 168-175]; cf. Lane, Arabic-English Lexicon, vol. 4, p. 1379f. and vol. 5, p. 1777. According to this source, the first word signifies "drop" or "leave out", and the second "strike" or "hit".

    77 Muqaddimah, trans. Rosenthal, vol. 3, p. 202 [III, 166].

[^20]:    78 Muqaddimah, trans. Rosenthal, vol. 3, p. 200, fn. 896.
    79 Muqaddimah, trans. Rosenthal, vol. 3, p. 202 [III,166]. The question mark is Rosenthal's, D.L.

[^21]:    80 Muqaddimah, trans. Rosenthal, vol. 3, p. 209 [III, 174].
    81 Muqaddimah, trans. Rosenthal, vol. 3, p. 206 [III, 171].
    82 Cf. Muqaddimah, trans. Rosenthal, vol. 3, p. 208, fn. 904. The abjad value of "h", however, is 5.
    83 Muqaddimah, trans. Rosenthal, vol. 3, p. 209 [III, 174].

[^22]:    86 This value is not used as a letter, but represents the "scale of the second growth", one (Muqaddimah, trans. Rosenthal, vol. 3, p. 205 [III, 170]).
    87 Muqaddimah, trans. Rosenthal, vol. 3, p. 206f. [III, 172]. By contrast, in the French translation the movement on the side of eight is often translated in an absolute manner: "On remonte au 5 sur le côté des 8" (Muqaddima, trans. Monteil, vol. 3, p. 1133).

[^23]:    88 The symbol reached by the previous 9 has to be omitted in counting.
    89 These calculations are only performed in a hypothetical way.
    90 At this place, empty fields are obviously counted.
    91 Cf. Mark Marino, Critical Code Studies. Electronic Book Review, April 2006. http://www.electronicbookreview.com/thread/electropoetics/codology.

[^24]:    92 Muqaddimah, trans. Rosenthal, vol. 3, p. 202 [III, 166].
    93 Muqaddimah, trans. Rosenthal, vol. 3, p. 205 [III, 170].
    94 Muqaddimah, trans. Rosenthal, vol. 3, p. 203f. [III, 168]. For the explanation of uss, see fn. 898. The same term already appeared in a different function before, see fn . 70. The previous retention of 50 from 52 in the seventh cycle possibly represents a similar procedure with the tens. As can be seen in the sixth section, where the uss of 54 is 4, the meaning of the expression cannot be "smallest divisor".
    95 Muqaddimah, trans. Rosenthal, vol. 3, p. 172 [III, 138].

[^25]:    98 Johannes Lohmann, Philosophie und Sprachwissenschaft (Berlin, 1965), p. 51f.
    99 Claude E. Shannon, A mathematical theory of communication. The Bell System Technical fournal 27 [3-4] (1948): 379-423 and 623-656, reprint pp. 1-55, quotation p. 15. On the lower redundancy of Semitic writing see Sampson, Writing Systems, pp. 92-94.
    100 In biblical Hebrew, רבע (RBA) means "to lie", רעב (RAB) "to starve", ברע (BRA) was the name of a king of Sodom, בער (BAR) "to put sth. away", ערב (ARB) "to exchange", עבר (ABR) "to overflow", while in English, the only two meaningful permutations of these three letters are "bar" and "bra"; cf. Wilhelm Gesenius, Hebrä̈sches und Aramäisches Handwörterbuch über das Alte Testament (Berlin, 1962). The reason for this is that the Semitic writing system clings less closely to the mouth, cf. D. Link, Chains to the West. Markov's theory of connected events and its transmission to Western Europe. Science in Context 19 [4] (2006): 561-589, esp. 583f. Unpronounceable letter combinations are almost impossible in Arabic.

[^26]:    101 In reverse, it has been suspected that "artificial intelligence" represented "Cargo Cult"-the senseless and archaic combination of technical objects not understood; cf. Richard Feynman, Cargo Cult Science. Engineering and Science 37 [7] (1974): 10-13; Kenneth Mark Colby, Modeling a paranoid mind. The Behavioural and Brain Sciences 4 (1981): 515-560, esp. 534.
    102 Siegfried Zielinski and Eckhard Fürlus, Introduction: Ars brevis umbrae et lucis, in: Variantology 3. On Deep Time Relations of Arts, Sciences and Technologies In China and Elsewhere, ed. S. Zielinski and E. Fürlus (Cologne, 2008), pp. 7-14, here p. 14.

    103 Danny Rubin and Harold Ramis, Groundbog Day (1993).
    104 Action movies from the 1990s, like "The Terminator" (James Cameron, 1984), "Total Recall" (Paul Verhoeven, 1990), "Terminator 2: Judgement Day" (James Cameron, 1991) and many others, playfully circle around this paradox.

[^27]:    105 Pierre-Simon Laplace, Pbilosophical Essay on Probabilities [1825], trans. Andrew I. Dale (New York, 1995), p. 6.

    106 If the decision was not taken beforehand, the demon would not dispose of all information that will determine the outcome.
    107 Muqaddimah, trans. Rosenthal, vol. 3, p. 171f. [III, 137f.].
    108 E. 7. Brill's First Encyclopedia of Islam, 1913-1936, ed. M. Th. Houtsma, vol. 7: S-Taiba (Leyden, 1987), p. 425 f. Also note the similarity of simiyā, the art of mixing verbal elements, and hīmiyā, alchemy (the processing of material ones), which immediately follows in Ibn Khaldūn's account.

[^28]:    113 Władysław Kozaczuk, Enigma. How the German Machine Cipher Was Broken, and How It Was Read by the Allies in World War Two, trans. Christopher Kasparek (Frederick, MD, 1984); see D. Link, Resurrecting Bomba Kryptologiczna: Archaeology of Algorithmic Artefacts, I. Cryptologia 33 [2] (2009): 166-182. The combination of the different cyclical components of the $z \bar{a} \bar{i}$ irjab is in fact quite similar to Enigma.
    114 Kahn, Codebreakers, p. 128; Arthur Scherbius, Ciphering machine (Patent no. US 1,657,411, filed in Germany as no. DE 383,594 on 12 February 1922).

[^29]:    115 Alan Turing, On computable numbers, with an application to the Entscheidungsproblem. Proceedings of the London Mathematical Society (Ser. 2) 42 (1936): 230-265. Any non-mechanisable investigation, on the other hand, usually fails at the complexity. Turing devised "oracles" capable of answering unsolvable questions; cf. A. Turing, Systems of logic based on ordinals. Proceedings of the London Mathematical Society (Ser. 2) 45 (1939): 161-228.
    116 E.W. Lane, An Account of the Manners and Customs of the Modern Egyptians, 2 vols. (London, 1836), vol. 1, p. 336 ff .

[^30]:    118 Al-Kadi has published the comparison of a frequency count he conducted with one that was performed more than one thousand years earlier by Al-Kindi; cf. Al-Kadi, Origins of cryptology, p. 112, Fig. 4.
    119 Muqaddimah, trans. Rosenthal, vol. 3, p. 171 [III, 137].

