



Thought Experiment

Juan-David Nasio's analysis (*Lacan: Topologically Speaking*) of the cross-cap as a comprehensive topology encompassing Lacan's theories about demand, desire, retroaction, *automaton*, and the *tuchē* or "encounter with the Real" concludes with the realization that the "disk" that is permissible at the lower portion of the cross-cap is actually a point — a *vanishing point*, a void.

If this is true, then in projective geometry every vanishing point is related to a "family" of parallel lines, but in this case the parallel lines in question *are those of the Möbius band* that is the signature of the upper region of the cross-cap. In this non-orientable surface, the appearance of two edges is indisputable; and, because they never "meet," it is equally clear that they are parallel. *Yet*, these edges are in fact a single line. As Nasio has insisted, the Möbius band can be reduced to a line, which is equivalent to the "interior-8."

1. Can one line be "read" as two lines? Yes, clearly, because we can grasp the Möbius band by its edges. Our fingers will never pinch together as they move across the full length of the band.
2. Are these lines parallel? Yes, clearly, because our fingers never pinch closed.
3. Is the edge each finger touches two edges or one? For the Möbius band, we have to answer "one," because of the twist that creates the 2-d surface that self-intersects and is non-oriented (meaning that this figure also lies within a "projective space," geometrically).
4. If, as Nasio argues, the disk that characterizes the lower portion of the cross-cap is really a point, a vanishing point that "lies at infinity," then the "two edges" must vanish at this point.
5. But, if the edge is really one line, then it, too, must vanish at this point, but "in two different directions."
6. Another way of seeing this is to say that one line vanishes at point X, and its other co-extensive dual vanishes at point Y, which are both the same and different vanishing point.

Discussion. The ambiguity of the Möbius band is well known, thanks to the ease of construction. We can “see” it in 3-space, but the question is, what is it that we see? A shape? A form? A material presence?” We do not see the twist. Although we see that the strip is *twisted*, we cannot see that the twist is, unlike what we see from any given angle at any point in time, fully distributed throughout the full length of the band.

Because this distribution is invisible to us, the twist actually exists within a virtual space. This virtuality also conceals the actual geometry of the band, by which its two obvious sides “turn out to be” really one, and it’s two edges that we see and can touch, “turn out to be” really one. We require a demonstration to “certify” this one-ness, by which we confirm at the same time that the strip is a dual: both two and one, or as James Joyce put it, “twone.”

Nasio makes the important leap by associating the lower segment of the cross-cap with a sack or half-sphere, which can be flattened in 3-space to a disk. This disk can be shrunk to the size of a point, and the point can be placed “at infinity” to serve as a vanishing point.

Every vanishing point “serves” a family of parallel lines. No matter how dispersed these lines may be, in projective geometry they will all meet at this point on the horizon. This makes the entire lower half of the cross-cap, which seems spherical, a point at infinity with respect to the upper region, where there is only a 2-d self-intersecting, non-orientable surface.

By having a single shape that can be actualized in Euclidean 3-space that is also a “theoretical” surface in 2-d projective space, the important leap can be made to the projectivity of the edge(s) of the Möbius band — namely, that they/it vanish(es) to a point that represents Euclidean perspectival 3-d space. This point is the black hole that contains our perceptual world. The Real lies above it.

Conclusion? The issue condenses around the Möbius band’s central paradox. Put in terms of parallels, the two sides and two edges that exist in our Euclidean interrogation of the strip, are the same side and same edge, but our illusion cannot be dismissed. The two-ness resists dismissal. It would be more informative to say that the sides and edges *converge*, and that this convergence “happens at infinity.” From the Euclidean point of view, this is an impossibility; from a Lacanian position, it is the Real. The value of putting things this way is that it emphasizes the role of the *point*, the vanishing point. It expands as a disk, the disk becomes a sack, a spherical container, an enclosed space. Its continence, however, is complicated by the fact that it is simultaneously a portal, a conditional passageway. It is impossible to conceptualize this combination in terms of Euclidean geometry, but in projective geometry, it is a necessary conclusion. Like the man said, Impossible = Real. The Other Side of Euclid is Projectivity.

If this be error and upon me prov’d,
I never writ, nor no man ever lov’d.

(Shakespeare’s contribution to the puzzle)